Ferromagnetic Resonance (FMR) Excited By A Linearly-Polarized Electromagnetic Wave

Vadym Zayets
Tsukuba, Ibaraki, Japan
(Dated: October 20, 2025)

This is calculations of a ferromagnetic resonance (FMR) describing the magnetization dynamics of a ferromagnetic material illuminated by a linearly polarized electromagnetic wave. In this configuration, precession is excited exclusively by the right-hand circularly polarized component of the wave, whose magnetic field rotates synchronously with the natural precession. Conversely, the left-hand circularly polarized component, whose magnetic field rotates in the opposite direction, has no effect on the FMR. The main part of this work was completed on January 15, 2025.

I. CONDITIONS, ASSUMPTIONS, AXIS DIRECTIONS

(condition 1):The equilibrium magnetization is perpendicular to the plane (the z- direction). It is the case of a relatively thin ferromagnetic field, when the interfacial anisotropy overcomes the demagnetization field.

(condition 2): Oscillating Magnetic field is linearly polarized along the x-axis.

(condition 3): The bias magnetic field H_z and, therefore, the Larmor frequency ω_L do not depend either on precession angle θ or precession phase φ. It means that the variation of precession angle is small and there is no Kittel effect.

(approximation 1): The calculations are done ignoring the precession damping. Later, the precession damping is included as the damping torque.

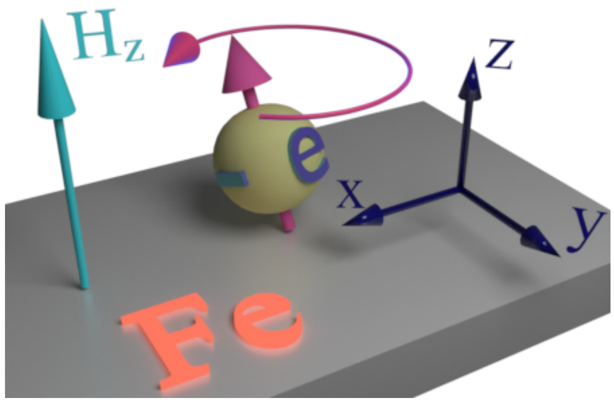

FIG. 1. Precession geometry. The equilibrium magnetization is perpendicular to the film (the z- direction). The external magnetic field H_z is applied along the easy axis (the z- direction). The precession is clockwise direction with respect to the external magnetic field (when to look from back to tip of the arrow)

II. FINAL RESULTS

During the magnetization precession around the z-axis, components of the magnetization are described as:

$$\begin{aligned} m_x(t) &= M \cdot \sin(\theta(t)) \cdot \cos(\omega_L t + \varphi(t)) \\ m_y(t) &= M \cdot \sin(\theta(t)) \cdot \sin(\omega_L t + \varphi(t)) \\ m_z(t) &= M \cdot \cos(\theta(t)) \end{aligned} \tag{1}$$

where m_x,m_y,m_z are component of magnetization M. The easy magnetic axis, and thus the precession axis, is aligned along the z-axis. The precession angle, θ, varies with time. In the case of ferromagnetic resonance (FMR), θ oscillates periodically around a relatively small average precession angle. In the case of parametric reversal, the average precession angle increases continuously until full magnetization reversal occurs. The precession frequency, ω_L, also varies with time. Typically, ω_L is larger for smaller precession angles θ and decreases as θ increases. The precession phase, φ, varies with time. When φ is in phase with the oscillations of the magnetic field of the pumping electromagnetic wave, the precession angle θ increases. When φ is out of phase, the precession angle θ decreases.

The magnetization precession is mathematically described by a solution to the Landau-Lifshitz equation for a system driven by an electromagnetic field of frequency ω. The temporal dynamics are governed by a set of two coupled differential equations. The first equation describes the torque-induced evolution of the precession angle θ, while the second governs the evolution of the precession phase φ. These equations (See Eq. 25), which must be solved simultaneously, are:

$$\begin{aligned} \frac{\partial\theta}{\partial t} &= 0.5 \cdot \Omega_{MW} \cdot \sin[(\omega - \omega_L)t - \varphi] \\ \frac{\partial\varphi}{\partial t} &= -0.5 \cdot \Omega_{MW} \frac{1}{\tan(\theta)} \cos[(\omega - \omega_L)t - \varphi] \end{aligned} \tag{2}$$

where $\omega_L = \gamma H_z$ is the Larmor frequency, $\Omega_{MW} = \gamma H_{MW}$ is the precession pumping strength and H_{MW} is the magnetic component of the pumping microwave electromagnetic field;

It is important to note that the dynamic equation is absolutely the same as in the case of a right- rotating magnetic field with half amplitude. Therefore, all results obtained for right- circularly- polarized microwave pump can be directly applied for the case of the linearly - polarized microwave pump

It is important to note that the solution (2) was obtained from Landau- Lifshitz equation without usage of any approximations. Set of differential equations Eqs. 2 is non-linear and should be solved either numerically or using some approximations.

III. SOLVING LL EQUATIONS. NO APPROXIMATIONS IN USE

The magnetization dynamic is calculated by solving the Landau-Lifshitz equation containing unchanged bias magnetic field H_z and the oscillating magnetic field H_{MW} of electromagnetic microwave, which excites the dynamics.

There is a bias perpendicular magnetic field $H_z = H_{ext} + H_{int}$, where H_{int} is the internal unchanged magnetic field and H_{ext} is the bias perpendicular magnetic field.

The Landau-Lifshitz (LL) equation describes the magnetization dynamic as :

$$\frac{\partial \vec{m}}{\partial t} = -\gamma \vec{m} \times (\vec{H} + \vec{H}_{MW}) \tag{3}$$

where γ is the electron gyromagnetic ratio, $\vec{H}$ is unchanged bias magnetic field and $\vec{H}_{MW}$ is oscillating magnetic field with the frequency ω (the magnetic component of the pumping microwave field), ω is the frequency of the microwave field, the microwave field is linear- polarized. The magnetic field the electromagnetic wave is in the plane of the spin precession (See Fig. **??**) :

$$\vec{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ H_z \end{pmatrix} \tag{4}$$

$$\vec{H}_{MW} = H_{MW} \begin{pmatrix} \cos(\omega t) \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\vec{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \tag{6}$$

Then

$$\vec{m} \times \vec{H} = H_z \begin{pmatrix} m_y \\ -m_x \\ 0 \end{pmatrix} \tag{7}$$

Then

$$\vec{m} \times \vec{H}_{MW} = H_{MW} \cos(\omega t) \begin{pmatrix} 0 \\ m_z \\ -m_y \end{pmatrix} \tag{8}$$

The scalar form of the Eq. 3 is

$$\begin{aligned} \tfrac{\partial m_x}{\partial t} &= -\omega_L m_y \\ \tfrac{\partial m_y}{\partial t} &= \omega_L m_x - \Omega_{MW} \cdot m_z \cos(\omega t) \\ \tfrac{\partial m_z}{\partial t} &= \Omega_{MW} m_y \cos(\omega t) \end{aligned} \tag{9}$$

where the Larmor frequency $\omega_L = \gamma H_z$, which is the precession frequency, and $\Omega_{MW} = \gamma H_{MW}$ is the precession pumping strength of the pumping microwave. It should be noted that ω_L is substantially larger than Ω_{MW}. If ω_L is typically around 10 GHz, Ω_{MW} is much smaller of about 1 MHz and less.

Introduction of new unknowns

$$\begin{aligned} m_+ &= m_x + i \cdot m_y & m_- &= m_x - i \cdot m_y \\ m_x &= \tfrac{m_+ + m_-}{2} & m_y &= \tfrac{m_+ - m_-}{2i} \end{aligned} \tag{10}$$

and addition/ subtraction of the 1st and 2nd equations of Eqs. (9) give

$$\begin{aligned} \tfrac{\partial m_+}{\partial t} &= i\omega_L m_- - i\Omega_{MW} \cdot m_z \cos(\omega t) \\ \tfrac{\partial m_-}{\partial t} &= -i\omega_L m_- + i\Omega_{MW} \cdot m_z \cos(\omega t) \\ \tfrac{\partial m_z}{\partial t} &= \Omega_{MW} \tfrac{m_+ - m_-}{2i} \cos(\omega t) \end{aligned} \tag{11}$$

The solution of Eq. 11 can be found in form of the spin precession, which can be described as

$$\begin{aligned} m_x(t) &= M \cdot \sin(\theta(t)) \cdot \cos(\phi(t)) \\ m_y(t) &= M \cdot \sin(\theta(t)) \cdot \sin(\phi(t)) \\ m_z(t) &= M \cdot \cos(\theta(t)) \end{aligned} \tag{12}$$

where $\theta(t)$ and $\phi(t)$ are new time- dependent unknowns and $M = \sqrt{m_x^2 + m_y^2 + m_z^2}$ is the magnetization, which is time-independent: $\frac{\partial M}{\partial t} = 0$.

Summing and substituting 1st and 2nd Eqs. of 12 gives:

$$\begin{aligned} m_+(t) &= M \cdot \sin(\theta(t)) \cdot e^{i\phi(t)} \\ m_-(t) &= M \cdot \sin(\theta(t)) \cdot e^{-i\phi(t)} \\ m_z(t) &= M \cdot \cos(\theta(t)) \end{aligned} \tag{13}$$

Differentiating Eqs. 13 gives

$$\begin{aligned} \tfrac{\partial m_+}{\partial t} &= M \cdot e^{i\phi} \left(\cos(\theta) \tfrac{\partial \theta}{\partial t} + i \tfrac{\partial \phi}{\partial t} \sin(\theta) \right) \\ \tfrac{\partial m_-}{\partial t} &= M \cdot e^{-i\phi} \left(\cos(\theta) \tfrac{\partial \theta}{\partial t} - i \tfrac{\partial \phi}{\partial t} \sin(\theta) \right) \\ \tfrac{\partial m_z}{\partial t} &= M \cdot \left(-\sin(\theta) \tfrac{\partial \theta}{\partial t} \right) \end{aligned} \tag{14}$$

Substitution of Eqs. 14, 13 into Eq. 11 and dividing over M gives

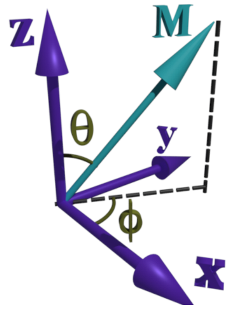

FIG. 2. Precession geometry. θ is the precession angle of magnetization M with respect to the easy axis (the z- axis). The ϕ is the precession phase with respect to the x- axis.

$$\begin{array}{l} \left[\cos(\theta)\frac{\partial\theta}{\partial t} + i\frac{\partial\phi}{\partial t}\sin(\theta)\right]e^{i\phi} = \\ = i\omega_L \sin(\theta)e^{i\phi} - i\Omega_{MW}\cos(\theta)\cos(\omega t) \\ \left[\cos(\theta)\frac{\partial\theta}{\partial t} - i\frac{\partial\phi}{\partial t}\sin(\theta)\right]e^{-i\phi} = \\ = -i\omega_L \sin(\theta)e^{-i\phi} + i\Omega_{MW}\cos(\theta)\cos(\omega t) \\ -\sin(\theta)\frac{\partial\theta}{\partial t} = \Omega_{MW}\cos(\omega t)\cdot sin(\theta)\frac{e^{+i\phi t}-e^{-i\phi t}}{2i} \end{array} \tag{15}$$

dividing the both sides of 1st equation over $\cos(\theta)e^{i\phi}$ and both sides of the 2nd equation over $\cos(\theta)e^{-i\phi}$ give

$$\begin{array}{l} \frac{\partial\theta}{\partial t} + i\frac{\partial\phi}{\partial t}\tan(\theta) = i\omega_L\tan(\theta) - i\Omega_{MW}\cos(\omega t)e^{-i\phi} \\ \frac{\partial\theta}{\partial t} - i\frac{\partial\phi}{\partial t}\tan(\theta) = -i\omega_L\tan(\theta) + i\Omega_{MW}\cos(\omega t)e^{+i\phi} \\ \frac{\partial\theta}{\partial t} = -\Omega_{MW}\cos(\omega t)\sin(\phi) \end{array} \tag{16}$$

It is important to note that only two of three equations of (Eq. 16) are independent. It is because summing the 1st and 2nd equations gives the 3rd equation.

The 3rd equation and subtraction of the 1st and 2nd equations gives the following system of two differential equations:

$$\begin{array}{l} i\frac{\partial\phi}{\partial t}\tan(\theta) = \\ = i\omega_L\tan(\theta) + i\cdot 0.5\cdot\Omega_{MW}\cos(\omega t)\left[-e^{-i\phi} - e^{+i\phi}\right] \\ \frac{\partial\theta}{\partial t} = -\Omega_{MW}\cos(\omega t)\sin(\phi) \end{array} \tag{17}$$

Dividing the 1st equation over $i\cdot\tan(\theta)$ gives the solution of LL equations as

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = \omega_L - \Omega_{MW}\frac{1}{\tan(\theta)}\cos(\omega t)\cos(\phi) \\ \frac{\partial\theta}{\partial t} = -\Omega_{MW}\cos(\omega t)\sin(\phi) \end{array} \tag{18}$$

Since

$$\begin{array}{l} \cos(x)\cos(y) = 0.5\left[\cos(x-y) + \cos(x+y)\right] \\ \cos(x)\sin(y) = 0.5\left[-\sin(x-y) + \sin(x+y)\right] \end{array} \tag{19}$$

Eq. (18) becomes

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = \omega_L - \frac{\Omega_{MW}}{2\cdot\tan(\theta)}\left[\cos(\omega t-\phi) + \cos(\omega t+\phi)\right] \\ \frac{\partial\theta}{\partial t} = \frac{\Omega_{MW}}{2}\left[\sin(\omega t-\phi) - \sin(\omega t+\phi)\right] \end{array} \tag{20}$$

It is important to note that the solution (18) was obtained from LL equations (3) without usage of any approximations

In the absence of the oscillating magnetic field $H_{MW} = 0$ (absence of the microwave pump), which leads to and $\Omega_{MW} = 0$, Eq.20 becomes

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = \omega_L \\ \frac{\partial\theta}{\partial t} = 0 \end{array} \tag{21}$$

which has a solution

$$\begin{array}{l} \phi = \omega_L t + \phi_0 \\ \theta = \theta_0 \end{array} \tag{22}$$

It describes the magnetization precession at a constant angle θ_0 at the Larmor frequency ω_L

Introduction of new independents as

$$\begin{array}{l} \phi = \omega_L t + \varphi(t) \\ \theta = \theta_0 + \theta_1(t) \end{array} \tag{23}$$

simplifies Eq. 20 as

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = -\frac{\Omega_{MW}}{2\cdot\tan(\theta_0+\theta_1(t))}\cdot \\ \cdot\left[\cos((\omega-\omega_L)t - \varphi(t)) + \cos((\omega+\omega_L)t + \varphi(t))\right] \\ \frac{\partial\theta}{\partial t} = \frac{\Omega_{MW}}{2}\cdot \\ \cdot\left[\sin((\omega-\omega_L)t - \varphi(t)) - \sin((\omega+\omega_L)t + \varphi(t))\right] \end{array} \tag{24}$$

The right side of each equation consists of two parts. The first part describes the oscillation of torque at low frequency $\omega - \omega_L$, while the second part describes the oscillation of torque at high frequency $\omega + \omega_L$, , which is approximately twice the FMR frequency ω_L. Each torque periodically forces the precession angle θ to increase and decrease. However, because the second torque oscillates over an extremely short period, the change in the precession angle, which it causes, is negligible. Therefore, the second torque can be neglected.

Ignoring in Eqs.(24) the parts oscillating with double frequency $\omega + \omega_L$, the solution is obtained as

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = -\frac{\Omega_{MW}}{2\cdot\tan(\theta_0+\theta_1(t))}\cos((\omega-\omega_L)t - \varphi(t)) \\ \frac{\partial\theta}{\partial t} = \frac{\Omega_{MW}}{2}\sin((\omega-\omega_L)t - \varphi(t)) \end{array} \tag{25}$$

This is the final solution. The 2nd Eq. describes the torque inserted by the microwave on spin precession. The torque forces the precession angle θ either to increase or to decrease. The 1st equation describes the modulation of the precession phase.

It is important to note that the dynamic equation is absolutely the same as in the case of a right- rotating magnetic field with half amplitude. Therefore, all results obtained for right- circularly- polarized microwave pump can be directly applied for the case of the linearly - polarized microwave pump