Inability to Excite Ferromagnetic Resonance by a Circularly Polarized Wave with Opposite Polarization Rotation to Magnetization Precession

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This work presents a calculation of ferromagnetic resonance (FMR), describing the magnetization dynamics of a ferromagnetic material under illumination by an electromagnetic wave. The analysis focuses on the case where the magnetic field component of the wave rotates in the direction opposite to the natural spin precession. In this configuration, the magnetization precession is not excited, and no resonance response occurs. There is a weak torque that oscillates at double the Larmor frequency, has a negligible effect on the magnetization and does not contribute to ferromagnetic resonance (FMR).The main part of this work was completed on January 15, 2025.

I. CONDITIONS, ASSUMPTIONS, AXIS DIRECTIONS

(condition 1):The equilibrium magnetization is perpendicular to the plane (the z- direction). It is the case of a relatively thin ferromagnetic field, when the interfacial anisotropy overcomes the demagnetization field.

(condition 2): Oscillating Magnetic field is circular polarized, and its polarization is rotating in the xy-plane opposite to the magnetization rotation.

(approximation 1): The calculations are done ignoring the precession damping.

II. FINAL RESULTS

In this case, the torque oscillates at twice the Larmor frequency. As a result, its polarity reverses within an extremely short time—far too rapidly to produce any noticeable change in the precession angle. Therefore, the time-averaged torque has no net effect on the magnetization precession and can be neglected.

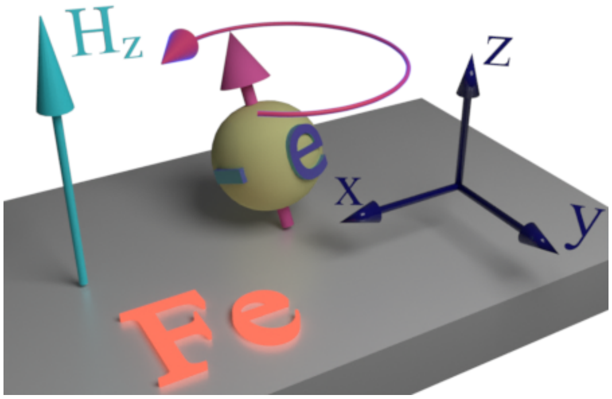

FIG. 1. Precession geometry. The equilibrium magnetization is perpendicular to the film (the z- direction). The external magnetic field H_z is applied along the easy axis (the z- direction). The precession is clockwise direction with respect to the external magnetic field (when to look from back to tip of the arrow). The rotation of magnetic field of the electromagnetic field is in the opposite direction (the counterclockwise direction)

The torque is calculated (See Eq.23) as

$$\boxed{\frac{\partial \theta_1}{\partial t} = -\Omega_{MW} \cdot \sin[(\omega + \omega_L)t - \varphi(t)]} \tag{1}$$

where where $\omega_L = \gamma H_z$ is the Larmor frequency, $\Omega_{MW} = \gamma H_{MW}$ is the precession pumping strength and H_{MW} is the magnetic component of the pumping microwave electromagnetic field; θ is the magnetization angle (precession angle) with respect to the easy axis and φ is phase difference between magnetization precession and the oscillating magnetic field, γ is the electron gyromagnetic ratio.

The phase of the torque is oscillating as (see Eq. 23)

$$\boxed{\frac{\partial \varphi}{\partial t} = -\frac{\Omega_{MW} \cdot}{\tan(\theta_0 + \theta_1)} \cos[(\omega + \omega_L)t - \varphi(t)]} \tag{2}$$

III. SOLVING LL EQUATIONS. NO APPROXIMATIONS IN USE

The magnetization dynamic is calculated by solving the Landau-Lifshitz equation containing unchanged bias magnetic field H_z and the oscillating magnetic field H_{MW} of electromagnetic microwave, which excites the dynamics.

There is a bias perpendicular magnetic field $H_z = H_{ext} + H_{int}$, where H_{int} is the internal unchanged magnetic field and H_{ext} is the bias perpendicular magnetic field.

The Landau-Lifshitz (LL) equation describes the magnetization dynamic as :

$$\frac{\partial \vec{m}}{\partial t} = -\gamma \vec{m} \times (\vec{H} + \vec{H}_{MW}) \tag{3}$$

where γ is the electron gyromagnetic ratio, $\vec{H}$ is unchanged bias magnetic field and $\vec{H}_{MW}$ is oscillating magnetic field with the frequency ω (the magnetic component

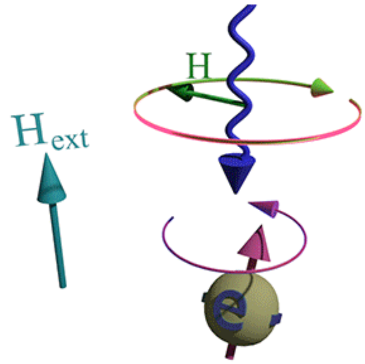

FIG. 2. Geometry of illuminating microwave field. The plane of polarization rotation (the xy-plane) is the same as the plane of the spin precession. The polarization rotation (counter-clockwise direction with respect to the external magnetic field H_z) is opposite to the direction of the spin precession (the clockwise direction).

of the pumping microwave field), ω is the frequency of the microwave field, the microwave field is circular- polarized with magnetic field circulary rotating in the opposite direction as the direction of spin precession (See Fig. 2) :

$$\vec{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ H_z \end{pmatrix} \tag{4}$$

$$\vec{H}_{MW} = H_{MW} \begin{pmatrix} \cos(\omega t) \\ -\sin(\omega t) \\ 0 \end{pmatrix} \tag{5}$$

$$\vec{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \tag{6}$$

Then

$$\vec{m} \times \vec{H} = H_z \begin{pmatrix} m_y \\ -m_x \\ 0 \end{pmatrix} \tag{7}$$

Then

$$\vec{m}\times\vec{H}_{MW} = H_{MW} \left[-\sin(\omega t) \begin{pmatrix} -m_z \\ 0 \\ m_x \end{pmatrix} + \cos(\omega t) \begin{pmatrix} 0 \\ m_z \\ -m_y \end{pmatrix} \right] \tag{8}$$

The scalar form of the Eq. 3 is

$$\begin{aligned} \tfrac{\partial m_x}{\partial t} &= -\omega_L m_y - \Omega_{MW} \cdot m_z \sin(\omega t) \\ \tfrac{\partial m_y}{\partial t} &= \omega_L m_x - \Omega_{MW} \cdot m_z \cos(\omega t) \\ \tfrac{\partial m_z}{\partial t} &= \Omega_{MW} \left[m_y \cos(\omega t) + m_x sin(\omega t) \right] \end{aligned} \tag{9}$$

where the Larmor frequency $\omega_L = \gamma H_z$, which is the precession frequency, and $\Omega_{MW} = \gamma H_{MW}$ is the precession pumping strength of the pumping microwave. ω_L is typically around 10 GHz, Ω_{MW} is much smaller of about 1 MHz and less.

Introduction of new unknowns

$$\begin{matrix} m_+ = m_x + i \cdot m_y & m_- = m_x - i \cdot m_y \\ m_x = \frac{m_+ + m_-}{2} & m_y = \frac{m_+ - m_-}{2i} \end{matrix} \tag{10}$$

and addition/ subtraction of the 1st and 2nd equations of Eqs. (9) give

$$\begin{aligned} \tfrac{\partial m_+}{\partial t} &= i\omega_L m_+ + \Omega_{MW} \cdot m_z \left[-\sin(\omega t) - i \cdot \cos(\omega t) \right] \\ \tfrac{\partial m_-}{\partial t} &= -i\omega_L m_- + \Omega_{MW} \cdot m_z \left[-\sin(\omega t) + i \cdot \cos(\omega t) \right] \\ \tfrac{\partial m_z}{\partial t} &= \Omega_{MW} \left[\tfrac{m_+ - m_-}{2i} \cos(\omega t) + \tfrac{m_+ + m_-}{2} \sin(\omega t) \right] \end{aligned} \tag{11}$$

or

$$\begin{aligned} \tfrac{\partial m_+}{\partial t} &= i\omega_L m_+ - i \cdot \Omega_{MW} \cdot m_z e^{-i\cdot\omega t} \\ \tfrac{\partial m_-}{\partial t} &= -i\omega_L m_- + i \cdot \Omega_{MW} \cdot m_z e^{+i\cdot\omega t} \\ \tfrac{\partial m_z}{\partial t} &= \tfrac{\Omega_{MW}}{2i} \left[m_+ e^{+i\cdot\omega t} - m_- e^{-i\cdot\omega t} \right] \end{aligned} \tag{12}$$

The solution of Eq. 12 can be found in form of the spin precession, which can be described as

$$\begin{aligned} m_x(t) &= M \cdot \sin(\theta(t)) \cdot \cos(\phi(t)) \\ m_y(t) &= M \cdot \sin(\theta(t)) \cdot \sin(\phi(t)) \\ m_z(t) &= M \cdot \cos(\theta(t)) \end{aligned} \tag{13}$$

where $\theta(t)$ and $\phi(t)$ are new time- dependent unknowns and $M = \sqrt{m_x^2 + m_y^2 + m_z^2}$ is the magnetization, which is time-independent: $\frac{\partial M}{\partial t} = 0$.

Summing and substituting 1st and 2nd Eqs. of 13 gives:

$$\begin{aligned} m_+ (t) &= M \cdot \sin\left(\theta\left(t\right)\right) \cdot e^{i\phi(t)} \\ m_- (t) &= M \cdot \sin\left(\theta\left(t\right)\right) \cdot e^{-i\phi(t)} \\ m_z (t) &= M \cdot \cos\left(\theta\left(t\right)\right) \end{aligned} \tag{14}$$

Differentiating Eqs. 14 gives

$$\begin{aligned} \tfrac{\partial m_+}{\partial t} &= M \cdot e^{i\phi} \left(\cos(\theta) \tfrac{\partial \theta}{\partial t} + i \tfrac{\partial \phi}{\partial t} \sin(\theta) \right) \\ \tfrac{\partial m_-}{\partial t} &= M \cdot e^{-i\phi} \left(\cos(\theta) \tfrac{\partial \theta}{\partial t} - i \tfrac{\partial \phi}{\partial t} \sin(\theta) \right) \\ \tfrac{\partial m_z}{\partial t} &= M \cdot \left(-\sin(\theta) \tfrac{\partial \theta}{\partial t} \right) \end{aligned} \tag{15}$$

Substitution of Eqs. 15, 14 into Eq. 12 and dividing over M gives

$$\begin{aligned} &\left[\cos(\theta) \tfrac{\partial \theta}{\partial t} + i \tfrac{\partial \phi}{\partial t} \sin(\theta) \right] e^{i\phi} = \\ &= i\omega_L \sin(\theta) e^{i\phi} - i \cdot \Omega_{MW} \cdot \cos(\theta) e^{-i\cdot\omega t} \\ &\left[\cos(\theta) \tfrac{\partial \theta}{\partial t} - i \tfrac{\partial \phi}{\partial t} \sin(\theta) \right] e^{-i\phi} = \\ &= -i\omega_L \sin(\theta) e^{-i\phi} + i \cdot \Omega_{MW} \cdot \cos(\theta) e^{+i\cdot\omega t} \\ &- \sin(\theta) \tfrac{\partial \theta}{\partial t} = \tfrac{\Omega_{MW}}{2i} sin(\theta) \left[e^{+i\cdot\phi t} e^{+i\cdot\omega t} - e^{+i\cdot\phi t} e^{+i\cdot\omega t} \right] \end{aligned} \tag{16}$$

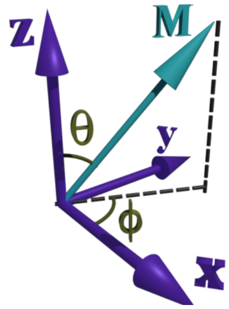

FIG. 3. Precession geometry. θ is the precession angle of magnetization M with respect to the easy axis (the z- axis). The ϕ is the precession phase with respect to the x- axis.

dividing the both sides of 1st equation over $\cos(\theta)e^{i\phi}$ and both sides of the 2nd equation over $\cos(\theta)e^{-i\phi}$ give

$$\begin{array}{l} \frac{\partial\theta}{\partial t} + i\frac{\partial\phi}{\partial t}\tan(\theta) = i\omega_L\tan(\theta) - i\cdot\Omega_{MW}\cdot e^{-i\cdot(\omega t+\phi)} \\ \frac{\partial\theta}{\partial t} - i\frac{\partial\phi}{\partial t}\tan(\theta) = -i\omega_L\tan(\theta) + i\cdot\Omega_{MW}\cdot e^{i\cdot(+\omega t+\phi)} \\ \frac{\partial\theta}{\partial t} = -\Omega_{MW}\sin(\omega t+\phi) \end{array} \tag{17}$$

It is important to note that only two of three equations of (Eq. 17) are independent. It is because summing the 1st and 2nd equations gives the 3rd equation.

The 3rd equation and subtraction of the 1st and 2nd equations gives the following system of two differential equations:

$$\begin{array}{l} i\frac{\partial\phi}{\partial t}\tan(\theta) = \\ = i\omega_L\tan(\theta) - i\cdot 0.5\cdot\Omega_{MW}\left[e^{-i\cdot(\omega t+\phi)} + e^{+i\cdot(\omega t+\phi)}\right] \\ \frac{\partial\theta}{\partial t} = -\Omega_{MW}\sin(\omega t+\phi) \end{array} \tag{18}$$

Dividing the 1st equation over $i\cdot\tan(\theta)$ gives the solution of LL equations as

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = \omega_L - \Omega_{MW}\frac{1}{\tan(\theta)}\cos(\omega t+\phi) \\ \frac{\partial\theta}{\partial t} = -\Omega_{MW}\sin(\omega t+\phi) \end{array} \tag{19}$$

It is important to note that the solution (19) was obtained from LL equations (3) without usage of any approximations

In the absence of the oscillating magnetic field $H_{MW} = 0$ (absence of the microwave pump), which leads to and $\Omega_{MW} = 0$, Eq.19 becomes

$$\begin{array}{l} \frac{\partial\phi}{\partial t} = \omega_L \\ \frac{\partial\theta}{\partial t} = 0 \end{array} \tag{20}$$

which has a solution

$$\begin{array}{l} \phi = \omega_L t + \phi_0 \\ \theta = \theta_0 \end{array} \tag{21}$$

It describes the magnetization precession at a constant angle θ_0 at the Larmor frequency ω_L

Introducing new independents as

$$\begin{array}{l} \phi = \omega_L t + \varphi(t) \\ \theta = \theta_0 + \theta_1(t) \end{array} \tag{22}$$

simplifies Eq. 19 as:

$$\begin{array}{l} \frac{\partial\varphi}{\partial t} = -\Omega_{MW}\frac{1}{\tan(\theta_0+\theta_1(t))}\cos[(\omega+\omega_L)t - \varphi(t)] \\ \frac{\partial\theta_1}{\partial t} = -\Omega_{MW}\sin[(\omega+\omega_L)t - \varphi(t)] \end{array} \tag{23}$$

This is the final solution. The 2nd Eq. describes the torque inserted by the microwave on spin precession. It oscillates at the high frequency $\omega + \omega_L$, which is approximately twice the FMR frequency ω_L. This torque periodically forces the precession angle θ to increase and decrease. Because this occurs over an extremely short period,the change in the precession angle within a period is negligible. The net torque averages to zero.