Brillouin and Langevin functions

Langevin model

Model aim: The Langevin model describes the energy distribution of electrons of different spin direction (the spin distribution) in an external magnetic field

If θ is the angle between magnetic moment μ at H, the magnetic energy will be

$$E_m = -\mu \cdot H \cdot \cos(\theta) \quad (1)$$

The number of electrons of energy E_m follows the Boltzmann statistics

$$n(E_m) \sim e^{\frac{E_m}{kT}} \quad (2)$$

Number of electrons which magnetic moment between θ and θ +d θ, ϕ and ϕ +d ϕ,

$$dn(\theta,\phi) = A \cdot e^{\frac{-\mu \cdot H \cdot \cos(\theta)}{kT}} \cdot d\theta \cdot d\phi \quad (3)$$

Integration over ϕ gives

$$dn(\theta) = A \cdot e^{\frac{-\mu \cdot H \cdot \cos(\theta)}{kT}} 2\pi \cdot \sin(\theta) d\theta \quad (4)$$

Eq. 4 is normalized to the total number n_d of d- electrons

$$n_d = 2\pi \cdot A \cdot \int_{-\pi}^{\pi} e^{\frac{-\mu \cdot H \cdot \cos(\theta)}{kT}} \sin(\theta) d\theta = 2\pi \cdot A \cdot \int_{-1}^{1} e^{\frac{-\mu \cdot H \cdot \cos(\theta)}{kT}} \mathrm{d}(\cos(\theta)) \quad (5)$$

Integration gives

$$n_d = \frac{4\pi \cdot A \cdot kT}{\mu \cdot H} \sinh\left(\frac{\mu \cdot H}{kT}\right) \quad (6)$$

Therefore, distribution of electrons over angles is

$$dn(\theta) = \frac{4\pi \cdot n_d \cdot kT}{\mu \cdot H} \sinh\left(\frac{\mu \cdot H}{kT}\right) \cdot e^{\frac{-\mu \cdot H \cdot \cos(\theta)}{kT}} 2\pi \cdot \sin(\theta) d\theta \quad (7)$$

The magnetic moments perpendicular to H will cancel on the average, and the total magnetic moment m along H is calculated as

$$m = \int_{-\pi}^{\pi} \mu \cdot \cos(\theta) dn(\theta) = A \cdot 2\pi \int_{-\pi}^{\pi} \mu \cdot \cos(\theta) e^{\frac{-\mu \cdot H \cdot \cos(\theta)}{kT}} \cdot \sin(\theta) d\theta \quad (8)$$

The integration by parts gives

$$\frac{M}{M_0} = L\left(\frac{\mu \cdot H}{kT}\right) \quad (9)$$

where M is the saturation magnetization and L(x) is the Langevin function

$$L(x) = \coth(x) - \frac{1}{x} \quad (11)$$

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}} \quad (12)$$

Ferromagnetism

Mean field. Weiss theory.

$$F = H + I \quad (20)$$

where F is the total magnetic field, I is the exchange field due to exchange interaction and the internal magnetic field, which is induced by the magnetic moments.

The exchange field I is changed proportionally to the change of magnetization

$$\frac{I}{I_0} = \frac{M}{M_0} \quad (21)$$

where I_0 is the exchange field at saturated magnetic moment.

Substitution of Eqs.9,11 into Eq.21 gives

$$\frac{I}{I_0} = L\left(\frac{\mu \cdot F}{kT}\right) = \coth\left(\frac{\mu \cdot F}{kT}\right) - \frac{kT}{\mu \cdot F} \quad (22)$$

Substitution of Eqs.20 into Eq.22 gives

$$\frac{I}{I_0} = L\left(\frac{\mu \cdot (H + I)}{kT}\right) \quad (23)$$

Or without external field

$$\frac{I}{I_0} = L\left(\frac{\mu \cdot I}{kT}\right) \quad (24)$$

Curie temperature
It is the case when the magnetization becomes zero M=0 and therefore the exchange field becomes zero as well I=0

Since

$$\lim_{x\to 0}\left(L(x)\right)=\frac{1}{3}x \quad (25)$$

Eq. 24 can be transformed at I->0

$$\lim_{x\to 0}\left[L(x)\right]=\frac{e^x+e^{-x}}{e^x-e^{-x}}-\frac{1}{x}=\frac{2}{2x+\frac{x^3}{3}}-\frac{1}{x}=\frac{1}{x}\left(\frac{2}{2+\frac{x^2}{3}}-1\right)=-\frac{1}{x}\frac{x^2/3}{2+x}=-\frac{x}{6} \quad (11)$$

Using Eq.24 at small I gives

$$\frac{I}{I_0}=\frac{1}{3}\frac{\mu\cdot I}{kT_C} \quad (26)$$

Where Tc is Curie temperature

$$\boxed{T_C=\frac{\mu\cdot I_0}{3k} \quad (26)}$$

As I is linearly proportional to M

$$I=I_0\frac{M}{M_0}=\frac{M}{M_0}\frac{3k\cdot T_c}{\mu} \quad (24)$$

$$\frac{M}{M_0}=L\left(\frac{\mu}{kT}\frac{M}{M_0}\frac{3k\cdot T_c}{\mu}\right)=L\left(3\frac{M}{M_0}\frac{T_c}{T}\right) \quad (24)$$

With external magnetic field

$$\frac{M}{M_0}=L\left(\frac{\mu\cdot(I+H)}{kT}\right)=L\left(\frac{\mu\cdot\left(\frac{M}{M_0}\frac{3k\cdot T_c}{\mu}+H\right)}{kT}\right)=L\left(3\frac{M}{M_0}\frac{T_c}{T}+H\frac{\mu}{kT}\right) \quad (23)$$

Derivative

$$\frac{\partial}{\partial H}\left(\frac{M}{M_0}\right)=\frac{\partial L}{\partial x}\left(x_0\right)\left[3\frac{T_c}{T}\frac{\partial}{\partial H}\left(\frac{M}{M_0}\right)+\frac{\mu}{kT}\right] \quad (23)$$

$$x_0=3\frac{M}{M_0}\frac{T_c}{T}+H\frac{\mu}{kT}$$

$$\frac{1}{M_0}\frac{\partial M}{\partial H}\left(1-3\frac{T_c}{T}\frac{\partial L}{\partial x}\right)=\frac{\mu}{kT}\frac{\partial L}{\partial x} \quad (23)$$