## Spin polarization of conduction electrons in a magnetic field, applied along spin- polarization of conduction electrons

The spin polarization  $P_S$  of an electron gas of conduction electrons increases, when a magnetic field is applied along spin direction of spin- polarized electrons. The increase is due to alignment of spins of spin- unpolarized electrons along the magnetic field. The dependence of the spin polarization  $P_S$  on a magnetic field H, which applied along the spin of the spin- polarized electrons, can be calculated as: Spin polarization of conduction electrons in a magnetic field,<br>applied along spin-polarization of conduction electrons<br>spin polarization P<sub>s</sub> of an electron gas of conduction electrons increases, when a<br>genetic field is a

$$
P_{S}(H) = \frac{P_{S,0} + \frac{H}{H_{S}}}{1 + \frac{H}{H_{S}}} \quad (S2.0)
$$

The case, when H is applied at an angle with respect to the spin of the spin- polarized electrons, is described here

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## Below

There is a spin precession of conduction electrons in a magnetic field. Additionally, the electron spin is aligned along the magnetic field due the Gilbert damming. The rate of the Gilbert damping is described in the Landau–Lifshitz (LL) equation by the Gilbert damping constant  $\gamma$ . The increase of the spin polarization due to the spin along the magnetic field can be calculated using rate equations.

The conversion of electrons from the group of spin polarized electrons to the group of spin- unpolarized electrons occurs due to misalignment spins of spin- polarized electrons from a single direction. The process is called the spin relaxation and is described by the spin relaxation time  $\tau_{relax}$ . The rate of the spin relaxation is proportional to the number of spin-polarized electrons and can be calculated as  $(H) = \frac{H}{H_S}$  (S2.0)<br>  $1 + \frac{H}{H_S}$ <br>
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$$
\frac{\partial n_{SU}}{\partial t} = \frac{n_{SP}}{\tau_{relax}} \quad (S2.1)
$$

where  $n_{SU}$  and  $n_{SU}$  are numbers of spin-unpolarized and spin-polarized conduction electrons, correspondingly.

The reverse process of the conversion of electrons from the group of spin unpolarized electrons to the group of spin- polarized electrons occurs due to alignment spins of spinpolarized electrons to a single direction, which is usually the spin direction of the localized electron. This process is called the spin pumping and the spin alignment is due to the sp-d exchange interaction between conduction and localized electrons, the scatterings between localized and the conduction electrons and the intrinsic magnetic field, which exists in a ferromagnetic along the magnetization direction. The rate of the spin pumping is described by the spin pumping time  $\tau_{\text{pump}}$ , is proportional to the number of spin-unpolarized electrons and can be calculated as and single directions occurs due to misalignment spins of spin-polarized electrons<br>an a single direction. The process is called the spin relaxation and is described by the<br>a relaxation time  $\tau_{relax}$ . The rate of the spin

$$
\frac{\partial n_{SP}}{\partial t} = \frac{n_{SU}}{\tau_{pump}} \quad (S2.2)
$$

The equilibrium numbers of spin- polarized and spin- unpolarized electrons are determined by the spin- relaxation and spin- pumping rates. In absence of a magnetic field, the balance condition gives e equilibrium numbers of spin- polarized and spin- unpolarized electrons are<br>termined by the spin- relaxation and spin- pumping rates. In absence of a magnetic<br>ld, the balance condition gives<br> $\frac{SP}{T_{pump}} = \frac{n_{SU}}{\tau_{pump}}$  (S2 the numbers of spin-polarized and spin-unpolarized electrons are<br>
2.3 by the spin-relaxation and spin-pumping rates. In absence of a magnetic<br>
alance condition gives<br>
(S2.3)<br>
2.3 Spin-polarization  $P_S$  is defined as the r e equilibrium numbers of spin-polarized and spin-unpolarized electrons are<br>
errmined by the spin-relaxation and spin-pumping rates. In absence of a magnetic<br>
id, the balance condition gives<br>  $\frac{SP}{d_{\text{g}}}= \frac{n_{\text{SJ}}}{\tau_{\text{$ 

$$
\frac{n_{SP}}{\tau_{relax}} = \frac{n_{SU}}{\tau_{pump}} \quad (S2.3)
$$

The spin polarization  $P<sub>S</sub>$  is defined as the ratio of number of spin-polarized electron to the sum of spin-polarized and spin- unpolarized electrons:

$$
P_{S} = \frac{n_{SP}}{n_{SP} + n_{SU}} \quad (S2.3a)
$$

From Eq. (S2.3) the spin polarization  $P_{S,0}$  in the absence of a magnetic field is calculated as

$$
P_{S,0} = P_S(H=0) = \frac{\tau_{relax}}{\tau_{relax} + \tau_{pump}} \quad (S2.4)
$$

When a magnetic field is applied along the spin direction of the spin polarized electrons, the spin pumping rate increases due to an additional mechanism of spin alignment for the spin- unpolarized electrons. This additional spin pumping mechanism can be described by additional spin pumping time  $\tau_{\text{pump,H}}$  and therefore the total spin pumping rate is calculated as s defined as the ratio of number of spin- polarized electron to<br>and spin- unpolarized electrons:<br>(b)<br>dolarization  $P_{S,\theta}$  in the absence of a magnetic field is calculated<br> $\frac{rel_{\theta}}{+ \tau_{pump}}$  (S2.4)<br>applied along the spin =  $P_s$  ( $H = 0$ ) =  $\frac{\tau_{relax}}{\tau_{relax} + \tau_{pump}}$  (S2.4)<br>
n a magnetic field is applied along the spin direction of the spin polarized electrons,<br>
pin pumping rate increases due to an additional mechanism of spin alignment for th

$$
\frac{n_{SP}}{\tau_{relax}} = \frac{n_{SU}}{\tau_{pump}} + \frac{n_{SU}}{\tau_{pump,H}} \quad (S2.5)
$$

The spin pumping rate due to spin alignment along the magnetic field was calculated in ref.[22] from a rigorous solution of the LL equation. In the case when

$$
t_{pump,H} \gg t_{scat} \quad (S2.6)
$$

The spin pumping time due to the spin alignment along the magnetic field can be calculated as

$$
t_{pump,H} = \frac{3}{2 \cdot g \cdot \lambda \cdot \mu_B \cdot H} \quad (S2.7)
$$

where g is the g-factor,  $\lambda$  is a phenomenological damping parameter of LL equation, and is  $\mu_B$  the Bohr magneton and  $t_{scat}$  is the average electron scattering time.

In the case of a temperature above the cryogenic temperatures and a moderate or low magnetic field, the condition (S2.6) is usually satisfied.

Since the damping term in the LL equation is linearly proportional to the magnetic field, the linear proportionality of the alignment rate (Eq.2.7) to the magnetic field is as expected.

In absence of a magnetic field, the balance condition gives

 $(S2.9)$ <br>er of spin-polarized electrons is calculated as<br> $\begin{bmatrix} S2.10 \end{bmatrix}$ relax  $\mathcal{U}_{pump}$   $\mathcal{U}_{pump,H}$  $\frac{n_{SP}}{2.9} = \frac{n_{SU}}{2.9} + \frac{n_{SU}}{2.9}$  (S2.9)  $\frac{\tau}{\tau}_{relax} - \frac{\tau}{\tau}_{pump} + \frac{\tau}{\tau}_{pup}$  $=\frac{n_{SU}}{n}$  + -From Eq.(S2.9), the number of spin- polarized electrons is calculated as +  $\frac{n_{SU}}{\tau_{pump,H}}$  (S2.9)<br>
2), the number of spin- polarized electrons is calculated as<br>  $\left(\frac{r_{\text{elax}}}{H_{pump}} + \frac{H}{H_{pump}}\right)$  (S2.10)<br>
=  $\frac{3}{2 \cdot g \cdot \lambda \cdot \mu_B \cdot \tau_{\text{relax}}}$  (S2.11)  $\begin{pmatrix} \tau_{\text{rel}} & H \end{pmatrix}$  $\frac{n_{\rm SP}}{\tau_{relax}} = \frac{n_{\rm SU}}{\tau_{pump}} + \frac{n_{\rm SU}}{\tau_{pump,H}} \quad (S2.9)$ <br>
From Eq.(S2.9), the number of spin-polarized electrons is calculated as<br>  $n_{\rm SP} = n_{\rm SU} \left( \frac{\tau_{\rm relax}}{\tau_{pump}} + \frac{H}{H_{pump}} \right) \quad (S2.10)$ <br>
where  $H_{pump} = \frac{3}{2 \cdot g \cdot \lambda \cdot \mu_B \cdot$ 

$$
n_{SP} = n_{SU} \left( \frac{\tau_{relax}}{\tau_{pump}} + \frac{H}{H_{pump}} \right) (S2.10)
$$
  
where  $H_{pump} = \frac{3}{2 \cdot g \cdot \lambda \cdot \mu_B \cdot \tau_{relax}}$  (S2.11)

spin polarization  $P_S$  in as a function of a magnetic field is calculated as

$$
\frac{n_{SP}}{\tau_{relax}} = \frac{n_{SU}}{\tau_{pump}} + \frac{n_{SU}}{\tau_{pump,H}}
$$
 (S2.9)  
\nFrom Eq.(S2.9), the number of spin-polarized electrons is calculated as  
\n
$$
n_{SP} = n_{SU} \left( \frac{\tau_{relax}}{\tau_{pump}} + \frac{H}{H_{pump}} \right)
$$
 (S2.10)  
\nwhere  $H_{pump} = \frac{3}{2 \cdot g \cdot \lambda \cdot \mu_B \cdot \tau_{relax}}$  (S2.11)  
\nspin polarization P<sub>S</sub> in as a function of a magnetic field is calculated as  
\n
$$
P_{S}(H) = \frac{\frac{\tau_{relax}}{\tau_{pump}} + \frac{H}{H_{pump}}}{1 + \frac{\tau_{relax}}{\tau_{pump}} + \frac{H}{H_{pump}}} = P_{S,0} \frac{1 + \frac{H}{H_{pump}} - \frac{1-P_{S,0}}{P_{S,0}}}{1 + \frac{H}{H_{pump}}} = P_{S,0} \frac{1 + \frac{H}{H_{S}} - \frac{1}{P_{S,0}}}{1 + \frac{H}{H_{pump}}}.
$$
 (S2.12)  
\nwhere  $H_S$  is the scaling field calculated as  
\n
$$
H_S = \frac{H_{pump}}{1 - P_{S,0}} = \frac{3}{2 \cdot g \cdot \lambda \cdot \mu_B \cdot \tau_{relax}} (1 - P_{S,0})
$$
 (S2.13)  
\nFigure 1 shows dependence of spin polarization  $P_S$  on magnetic field H. The  $P_S$  increase is  
\ninverse is

where  $H<sub>S</sub>$  is the scaling field calculated as

$$
H_{S} = \frac{H_{pump}}{1 - P_{S,0}} = \frac{3}{2 \cdot g \cdot \lambda \cdot \mu_{B} \cdot \tau_{relax} (1 - P_{S,0})}
$$
 (S2.13)

Figure 1 shows dependence of spin polarization  $P<sub>S</sub>$  on magnetic field H. The  $P<sub>S</sub>$ monolithically increases from  $P_{S,0}$  to 100%. The smaller the  $P_{S,0}$  is, the stepper the increase is.



Fig.1 Spin polarization  $P_s$  as a function of external magnetic field H. (a),  $H_s$ =7 kGauss,  $P_{s,0}$  is a parameter (b)  $P_{S,0} = 25\%$ , H<sub>S</sub> is parameter

In order to distinguish a magnetic effect, which depends on spin polarization or the same on the magnetic moment of conduction electrons (i.e. the Inverse Spin Hall effect), from magnetic effects, which depends on external magnetic field H (i.e. the ordinary Hall effect) or the magnetic moment of the localized electrons M (i.e. the Anomalous Hall effect), a steeper change of  $P<sub>S</sub>$  vs H is important. Figure 2 shows dependence of the 1st derivation on magnetic field H. The 1st is larger and stepper for a smaller  $P_{S,0}$  and smaller  $H<sub>S</sub>$ .



Fig.2 1st order derivative of P<sub>s</sub> as a function of external magnetic field H. (a),  $H_S$ =7 kGauss, P<sub>S,0</sub> is a parameter (b)  $P_{S,0} = 25\%$ , H<sub>S</sub> is parameter