Ambiguity for the evaluation of the AHE and ISHE contributions. Ambiguity for the evaluation of the spin polarization.

The Hall effect in a ferromagnetic metal is the sum of three contributions: (**contribution 1**) Ordinary Hall effect (OHE), which is proportional to an external magnetic field H; (**contribution 2**) Anomalous Hall effect (AHE), which is proportional to the magnetic moment of localized electrons M; and Inverse Spin Hall effect (ISHE), which is proportional to the magnetic moment of conduction electrons m;

In a ferromagnetic nanomagnet with the perpendicular magnetic anisotropy (PMA), the local magnetic moments are firmly fixed perpendicularly to plane and do not change with an external magnetic change. As a result, the AHE contribution in nanomagnet is a constant vs. H.

The ISHE contribution is proportional to the spin polarization of the conduction electrons $P_S = \dfrac{P_{S,0} + \dfrac{H}{H_S}}{1 + \dfrac{H}{H_S}}$

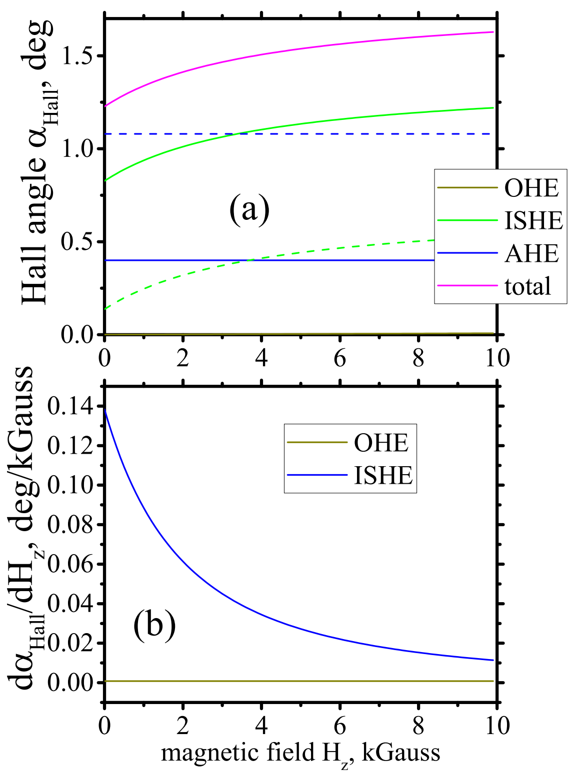

Fig.S1.1 Ambiguity of the data fitting. Two possible fittings: (solid lines) P_S=60 %, α_{ISHE}= 827 mdeg α_{AHE}= 400 mdeg (dash lines) P_S=20 %, α_{ISHE}= 137 mdeg α_{AHE}= 1080 mdeg. .H_S=3.985 kGauss and α_{OHE}=0.2 mdeg/kGauss for both cases. Both fitting give the identical total Hall angle (pink line) and its 1st derivation.

The dependence of Hall angle α_{Hall} on an external perpendicular magnetic field H in nanomagnet with PMA is described as

$$\alpha_{Hall} = \alpha_{OHE} \cdot H + \alpha_{AHE} + \alpha_{ISHE} \cdot \frac{P_{S,0} + \dfrac{H}{H_S}}{1 + \dfrac{H}{H_S}} \qquad (S1.1)$$

where α_{OHE} is the angle of the OHE contribution, α_{AHE} is the angle of the AHE contribution and α_{ISHE} is the angle of the ISHE contribution, $P_{S,0}$ is spin polarization at H=0 and H_S is the scaling magnetic field.

The ambiguity is originated from the fact that the functional dependence (S1) does not change when the set of three initial fitting parameters $\left(\alpha_{AHE} \quad \alpha_{ISHE} \quad P_{S,0}\right)$ changes to a new set $\left(\alpha^*_{AHE} \quad \alpha^*_{ISHE} \quad P^*_{S,0}\right)$, which is related to the initial set as

$$\alpha^*_{ISHE} = \alpha_{ISHE} \frac{1 - P_{S,0}}{1 - P^*_{S,0}} \frac{P^*_{S,0}}{P_{S,0}} \qquad (S1.2)$$

$$\alpha^*_{AHE} = \alpha_{AHE} + \alpha_{ISHE} \frac{P_S - P^*_S}{P_S\left(1 - P^*_S\right)} \quad (S1.3)$$

In order to prove this fact, a comparison of Eq. (S1.1) for two sets of parameters gives

$$\alpha_{AHE} + \alpha_{ISHE} \cdot \frac{1 + x\frac{1}{P_{S,0}}}{1+x} = \alpha^*_{AHE} + \alpha^*_{ISHE} \cdot \frac{1 + x\frac{1}{P^*_{S,0}}}{1+x} \quad (S1.4)$$

where $x = H / H_S$. Eq.(S1.4) is simplified as

$$\alpha_{AHE}\left(1+x\right) + \alpha_{ISHE} \cdot \left(1 + x\frac{1}{P_{S,0}}\right) = \alpha^*_{AHE}\left(1+x\right) + \alpha^*_{ISHE} \cdot \left(1 + x\frac{1}{P^*_{S,0}}\right) \quad (S1.5)$$

Comparison of the coefficients at x^0 gives

$$\alpha_{AHE} + \alpha_{ISHE} = \alpha^*_{AHE} + \alpha^*_{ISHE} \quad (S1.6)$$

Comparison the coefficients at x^1 gives

$$\alpha_{AHE} + \frac{\alpha_{ISHE}}{P_S} = \alpha^*_{AHE} + \frac{\alpha^*_{ISHE}}{P^*_S} \quad (S1.7)$$

Solution of Eqs. (S1.6),(S1.7) gives Eqs. (S1.2),(S1.3)

Figure S1.1 compares the OHE, ISHE, AHE contributions for two sets of parameters.