Neel model of magnetic anisotropy. The oversimplified classical model

L. Néel, Adv. Phys., 1955

Zayets 2018.11 (last modified 2021.11)

Content

This classical model is based on some oversimplified assumptions. This model ignores the specific features of the spin-orbit interaction, which creates the magnetic anisotropy. However, this model is a very simple and can be used for a rough estimate.

In this model, the total magnetic energy, which is a sum of the energy of the magnetic anisotropy E_{PMA} and the magnetic dipole energy can be calculated as

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M} \right)^2 + \vec{M} \cdot \vec{H} \quad (1a)$$

in which the energy of the magnetic anisotropy is assumed to be proportional to a square of the magnetic component M_z along the easy axis. Eq.(1a) can be written in the similar form as

$$-E = E_{PMA} \cdot \cos^2(\theta) + M \cdot H \cdot \cos(\theta - \phi) \quad (1)$$

where θ is the angle between the magnetization M and the film normal, φ is the angle between the magnetic field H and the film normal, EPMA is the energy of the perpendicular magnetic anisotropy, which includes the energy due to the demagnetization field.

Part 2. Equilibrium state. There is no an external magnetic field.

In the case H=0, the magnetic energy is calculated from Eq.(1) as

$$E = -E_{PMA} \cdot \cos^2(\theta) \quad (1.2)$$

Eq. (1.2) has two minimums at θ=0 deg and 180 deg corresponding to two equilibrium states when magnetization is parallel to the easy axis. The minimum energy equals E_{min}=-E_{PMA}.

Eq. (1.2) has two maximums at θ=+90 deg and -90 deg corresponding to two directions parallel to the hard axis. The maximum energy equals E_{max}=-0.

Part 3. In-plane magnetic field Hx. Calculation of anisotropy field H_{ani}

In this case there is only a component of the magnetic field H_x along the hard axis, H_z=0 and Eq.(1a) is simplified to

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M} \right)^2 + M_x \cdot H_x \quad (3)$$

or

$$-E = E_{PMA} \cdot \frac{M^2 - M_x^2}{M^2} + M_x \cdot H_x \quad (3a)$$

A stable state corresponds to the minimum of the energy, which can be found from the condition:

$$0 = -\frac{\partial E}{\partial M_x} = -2E_{PMA} \cdot \frac{M_x}{M^2} + H_x \quad (4)$$

Solution of Eq.(4) is

$$H_x = 2E_{PMA} \cdot \frac{M_x}{M^2} \quad (6a)$$

or

$$\boxed{\frac{M_x}{M} = \frac{H_x}{H_{ani}} \quad (6)}$$

where

$$H_{ani} = \frac{2E_{PMA}}{M} \quad (6b)$$

is the anisotropy files, which is the magnetic field, at which the magnetization M is aligned along the hard axis.

The anisotropy field H_{ani} linearly increases with an increase of PMA energy E_{PMA} . Since E_{PMA} is proportional to M^2 , (see Eq.31), H_{ani} is linearly proportional to magnetization M.

The linear dependence of Eq.(6) is important, often used and can be easily verified experimentally. That is the reason why the oversimplified Neel model was used for a long tome.
It should be noted that the correct model, which is used the properties of the spin- orbit interaction, gives the same linear dependence of Eq.(6)

Part 4 There are both H_x and H_z of the external magnetic field
In this case Eq.(1a) is

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M} \right)^2 + M_x \cdot H_x + M_z \cdot H_z \quad (1c)$$

or

$$-E = E_{PMA} \cdot \frac{M^2 - M_x^2}{M^2} + M_x \cdot H_x + H_z \cdot \sqrt{M^2 - M_x^2} \quad (9)$$

A stable state corresponds to the minimum of the energy, which can be found from the condition:

$$0 = -\frac{\partial E}{\partial M_x} = -2E_{PMA} \cdot \frac{M_x}{M^2} + H_x - H_z \cdot \frac{2M_x}{2\sqrt{M^2 - M_x^2}} \quad (10)$$

or

$$0 = -H_{ani0} \cdot \frac{M_x}{M} + H_x - H_z \cdot \frac{M_x}{\sqrt{M^2 - M_x^2}} \quad (10a)$$

where

$$H_{ani0} = \frac{2E_{PMA}}{M} \quad (6c)$$

is anisotropy field in absence of Hz.

$$H_x \frac{M}{M_x} = H_{ani0} + H_z \cdot \frac{M}{\sqrt{M^2 - M_x^2}} \quad (10b)$$

The solution of Eq.(10) is

$$\boxed{\frac{M_x}{M} = \frac{H_x}{H_{ani}} \quad (11)}$$

where

$$H_{ani} = H_{ani0} + H_z \cdot \frac{1}{\sqrt{1-\frac{M_x^2}{M^2}}} \quad (12)$$

Part 5. Energy barrier for the magnetization switching

In this the case when a magnetic field is applied along the magnetic easy axis (perpendicularly to the film) (φ=0), the energy is calculated from Eq.(1) as

$$-E = E_{PMA} \cdot \cos^2(\theta) + M \cdot H \cdot \cos(\theta) \quad (1c)$$

The maximums and minimums of energies can be found from the condition

$$0 = -\frac{\partial E}{\partial \theta} = -2E_{PMA} \cdot \cos(\theta)\sin(\theta) - M \cdot H \cdot \sin(\theta) \quad (20)$$

The energy maximum is at the magnetization angle θ_{max}

$$\cos(\theta_{\max}) = -\frac{M \cdot H}{2 \cdot E_{PMA}} \quad (21)$$

Substitution of Eq.(6b) into Eq.(21) gives

$$\cos(\theta_{\max}) = -\frac{H}{H_{ani}} \quad (21)$$

In the case when the external field H is substantially smaller than the anisotropy field H_{ani}, the magnetization angle of the maximum energy is closed to the hard axis direction θ_{max} ~ +90 deg and -90 deg.

Substitution of Eq. (21) into Eq.(1c) gives the maximum energy as

$$E_{\max} = -E_{PMA} \cdot \left(\frac{M \cdot H}{2 \cdot E_{PMA}}\right)^2 + M \cdot H \cdot \frac{M \cdot H}{2 \cdot E_{PMA}} = \frac{(M \cdot H)^2}{4 \cdot E_{PMA}} \quad (22)$$

The energy minimum is at θ min=0 and 180 degrees, the minimum energy is

$$E_{\min} = -E_{PMA} + M \cdot H \quad (23)$$

The energy barrier is a difference between the minim and maximum energy

$$E_{barrier} = E_{\max} - E_{\min} = \frac{(M \cdot H)^2}{4 \cdot E_{PMA}} - E_{PMA} + M \cdot H \quad (24)$$

or

$$E_{barrier} = E_{PMA}\left[\frac{(M \cdot H)^2}{4 \cdot E_{PMA}^2} - 1 + \frac{M \cdot H}{E_{PMA}}\right] \quad (24)$$

Eq.(6b) gives

$$E_{PMA} = \frac{H_{ani}M}{2} \quad (6c)$$

Substitution of Eq.6c into Eq.24 gives

$$E_{barrier} = E_{PMA}\left[\frac{H^2}{H_{ani}^2} - 1 + 2\frac{H}{H_{ani}}\right] = E_{PMA}\left[1 - \frac{H}{H_{ani}}\right]^2 \quad (25a)$$

$$\boxed{E_{barrier} = E_{PMA}\left[1 - \frac{H}{H_{ani}}\right]^2 \quad (25)}$$

The magnetization switching occurs at magnetic field equals to the coercive field H_c, which is substantially smaller that H_{ani}. **($H_c << H_{ani}$)** In this case

$$E_{barrier} \approx E_{PMA}\left(1 - \frac{2H}{H_{ani}}\right) \quad (26)$$

Substitution of Eq.6c into Eq.26 gives

$$\boxed{E_{barrier} \approx E_{PMA} + H \cdot M \quad (27)}$$

Note it is the energy barrier in the case when the magnetic field is along the easy axis and the magnetization direction is changing. It means that it is the case of the magnetization precession.

Part 6. Comparison of this oversimplified Neel model with the full model, which includes properties of the spin-orbit interaction

In the Neel model, the magnetic energy is described as

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M}\right)^2 + \vec{M} \cdot \vec{H} \quad (1a)$$

where M is the magnetization, H is external magnetic field M_z is the magnetization component along the easy axis and M_x is the magnetization component along the hard axis. Eq. 10.1 can be wrote as

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M}\right)^2 + M_z H_z + M_x H_x \quad (1c)$$

This the full model, which includes properties of the spin-orbit interaction, describes the magnetic energy as

$$-E = M_z^2 \left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1 \right) + \left(1+k_{so}\right) H_z M_z + H_x M_x + M^2 \quad (30)$$

Comparison of Eqs.(1c) and (30) gives

$$E_{PMA} = M^2 \left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1 \right) \quad (31)$$

From comparison of the 2nd and 3rd term, it can be concluded that the oversimplified Neel model are fully identical when the coefficient of the spin-orbit interaction equals to zero:

$$k_{so} = 0 \quad (32)$$

It is nearly the case for a single-layer ferromagnetic nanomagnet, but it is not the case for a multilayer nanomagnet.