The effective internal magnetic field in nanomagnet with perpendicular magnetic anisotropy (PMA)

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Case when **equilibrium magnetization of the nanomagnet is perpendicular-to-plane**

(Conditions) A very strong exchange interaction between all localized electrons. As a result, there are no magnetic domains and all spins are parallel each other

(used Fact) localized electrons at interface and in the bulk experience different magnetic field of the spin-orbit interaction. However, their spins are strongly bound together and all spin can be considered as one magnetic object, which experience one common effective field, which is called the internal magnetic field

(object): a nanomagnet, which easy axis is perpendicular-to-plane.

The magnetization is aligned along the direction of the total magnetic field, which is some of the internal and external magnetic field $H_{total}=H_{int}+H_{ext}$. Therefore, magnetization angle α_M with respect to film normal can be calculated as

$$\tan\left(\alpha_M\right)=\frac{M_{\|}}{M_{\perp}}=\frac{H_{\text{ext},\|}+H_{\text{int},\|}}{H_{\text{ext},\perp}+H_{\text{int},\perp}} \quad (1)$$

in equilibrium (in absence of external magnetic field) $\alpha_M=0$

Taking into account that the magnitude of the magnetization can be calculated as

$$M^2=M_{\perp}^2+M_{\|}^2 \quad (2)$$

The Eq.(1) is simplified as

$$\frac{\frac{M_{\|}}{M}}{\sqrt{1-\left(\frac{M_{\|}}{M}\right)^2}}=\frac{H_{\text{ext},\|}+H_{\text{int},\|}}{H_{\text{ext},\perp}+H_{\text{int},\perp}} \quad (3)$$

It assumed that the nanomagnet is sufficiently wide (in comparison to its thickness), so there is neither demagnetization field nor the spin-orbit field in the in--plane direction. As a result, the in-plane internal magnetic field has only one contribution of magnetic field induced by the magnetization, which is just the magnetic field along the magnetization:

$$H_{\text{int},\|} = M_{\|} \quad (4)$$

Substitution of Eq.(4) into (3) gives

$$\frac{\frac{M_{\|}}{M}}{\sqrt{1-\left(\frac{M_{\|}}{M}\right)^2}} = \frac{H_{\text{ext},\|} + M_{\|}}{H_{\text{ext},\perp} + H_{\text{int},\perp}} \quad (5)$$

The in--plane component of magnetization $M_{\|}$ is calculated from known (measured) anisotropy field H_{ani}. The feature of PMA and spin--orbit interaction is that the in--plane component of the magnetization linearly depends on the in--plane external magnetic field (see here) and the proportionality coefficient is defined as the anisotropy field:

$$\frac{M_{\|}}{M} = \frac{H_{\text{ext},\|}}{H_{ani}} \quad (6)$$

Substitution of Eq.(6) into Eq.(4) gives the in-plane component of the internal magnetic field as:

$$H_{\text{int},\|} = M\frac{H_{\text{ext},\|}}{H_{ani}} \quad (4a)$$

Substitution of Eq.(6) into Eq.(5) gives

$$\frac{\frac{H_{\text{ext},\|}}{H_{ani}}}{\sqrt{1-\left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2}} = \frac{H_{\text{ext},\|} + M\frac{H_{\text{ext},\|}}{H_{ani}}}{H_{\text{ext},\perp} + H_{\text{int},\perp}} \quad (7)$$

From Eq.(7) the perpendicular component of the internal magnetic field is found as

$$H_{\text{int},\perp} = \frac{H_{\text{ext},\|}\left(1+\frac{M}{H_{ani}}\right)}{\frac{H_{\text{ext},\|}}{H_{ani}}}\sqrt{1-\left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2} - H_{\text{ext},\perp} \quad (8)$$

or

$$\boxed{H_{\text{int},\perp} = \left(H_{ani}+M\right)\sqrt{1-\left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2} - H_{\text{ext},\perp} \quad (10)}$$

The angle α_{int} of the internal magnetic field with respect to film normal can be calculated as

$$\cot\left(\alpha_{H\,\text{int}}\right) = \frac{1}{\tan\left(\alpha_{H\,\text{int}}\right)} = \frac{H_{\text{int},\perp}}{H_{\text{int},\|}} = \frac{\left(H_{ani}+M\right)\sqrt{1-\left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2} - H_{\text{ext},\perp}}{H_{\text{int},\|}} \quad (11)$$

Substitution of Eq.(4a) into (11) gives

$$\cot\left(\alpha_{H\,\text{int}}\right) = \frac{\left(H_{ani}+M\right)\sqrt{1-\left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2} - H_{\text{ext},\perp}}{M\frac{H_{\text{ext},\|}}{H_{ani}}} \quad (12)$$

or

$$\boxed{\cot\left(\alpha_{H\,\text{int}}\right) = \left(1+\frac{H_{ani}}{M}\right)\sqrt{\left(\frac{H_{ani}}{H_{\text{ext},\|}}\right)^2 - 1} - \frac{H_{\text{ext},\perp}}{H_{\text{ext},\|}}\frac{H_{ani}}{M} \quad (12a)}$$

The angle α_M of the magnetization with respect to film normal can be calculated as

$$\cot\left(\alpha_M\right) = \frac{M_\perp}{M_\|} = \frac{\sqrt{1-\left(\frac{M_\|}{M}\right)^2}}{\frac{M_\|}{M}} = \frac{\sqrt{1-\left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2}}{\frac{H_{\text{ext},\|}}{H_{ani}}} \quad (14)$$

or

$$\boxed{\cot\left(\alpha_M\right) = \sqrt{\left(\frac{H_{ani}}{H_{\text{ext},\|}}\right)^2 - 1} \quad (15)}$$

Comparison of (12) and (14) gives the ratio between angles α_M and α_{Hint} as

$$\cot\left(\alpha_{H\,\mathrm{int}}\right)=\left(1+\frac{H_{ani}}{M}\right)\cot\left(\alpha_M\right)-\frac{H_{\mathrm{ext},\perp}}{H_{\mathrm{ext},\parallel}}\frac{H_{ani}}{M} \quad (16)$$

or

$$\boxed{\cot\left(\alpha_{H\,\mathrm{int}}\right)=\left(\frac{H_{ani}}{M}+1\right)\cot\left(\alpha_M\right)-\frac{H_{ani}}{M}\cot\left(\alpha_{Hext}\right) \quad (17)}$$

where the angle α_{Hext} of the angle of the external magnetic field, which is calculated as

$$\cot\left(\alpha_{Hext}\right)=\frac{H_{ext,\perp}}{H_{ext,\parallel}} \quad (18)$$

Magnitude of the internal magnetic field

$$H_{\mathrm{int}}=\sqrt{H_{\mathrm{int},\perp}^2+H_{\mathrm{int},\parallel}^2}=\sqrt{M_\parallel^2+\left[\left(H_{ani}+M\right)\sqrt{1-\left(\frac{H_{\mathrm{ext},\parallel}}{H_{ani}}\right)^2}-H_{\mathrm{ext},\perp}\right]^2} \quad (19)$$

or

$$H_{\mathrm{int}}=\sqrt{\left(M\frac{H_{\mathrm{ext},\parallel}}{H_{ani}}\right)^2+\left[\left(H_{ani}+M\right)\sqrt{1-\left(\frac{H_{\mathrm{ext},\parallel}}{H_{ani}}\right)^2}-H_{\mathrm{ext},\perp}\right]^2} \quad (20)$$

<u>The total magnetic field</u>

The total magnetic field is the sum of the internal and external magnetic fields.
From Eq.(4a)

$$H_{\mathrm{total},\parallel}=H_{\mathrm{int},\parallel}+H_{\mathrm{ext},\parallel}=H_{\mathrm{ext},\parallel}\left(1+\frac{M}{H_{ani}}\right) \quad (20)$$

from Eq.(10)

$$H_{\mathrm{total},\perp}=H_{\mathrm{int},\perp}+H_{\mathrm{ext},\perp}=\left(H_{ani}+M\right)\sqrt{1-\left(\frac{H_{\mathrm{ext},\parallel}}{H_{ani}}\right)^2} \quad (21)$$

The absolute value of the total field

$$H_{\text{total}} = \sqrt{H^2_{\text{total},\perp} + H^2_{\text{total},\|}} = \sqrt{\left(H_{ani} + M\right)^2 \left(1 - \left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2\right) + H^2_{\text{ext},\|}\left(1 + \frac{M}{H_{ani}}\right)^2} \quad (22)$$

$$H_{\text{total}} = \sqrt{\left(H_{ani} + M\right)^2 - H^2_{\text{ext},\|}\frac{\left(H_{ani} + M\right)^2}{H^2_{ani}} + H^2_{\text{ext},\|}\left(1 + \frac{M}{H_{ani}}\right)^2} = \sqrt{\left(H_{ani} + M\right)^2}$$

or

$$\boxed{H_{\text{total}} = \left(H_{ani} + M\right) \quad (23)}$$

Angle of the total magnetic field

$$\cot\left(\alpha_{H,total}\right) = \frac{H_{\text{total},\perp}}{H_{\text{total},\|}} = \frac{\left(H_{ani} + M\right)\sqrt{1 - \left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2}}{H_{\text{ext},\|}\left(1 + \frac{M}{H_{ani}}\right)} = \frac{\sqrt{1 - \left(\frac{H_{\text{ext},\|}}{H_{ani}}\right)^2}}{\frac{H_{\text{ext},\|}}{H_{ani}}} = \sqrt{\left(\frac{H_{ani}}{H_{\text{ext},\|}}\right)^2 - 1} \quad (24)$$

Comparison with Eq/(15) gives

$$\alpha_{H,total} = \alpha_M \quad (25)$$

The magnetization is aligned along the total magnetic field as was stated at the beginning.

Case when in-plane magnetic field is greater that the anisotropy field

In this case the magnetization is aligned in the in-plane direction

Therefore ethe total magnetic field can be expressed as

$$\boxed{H_{\text{total}} = \begin{matrix} H_{ani} + M & \left\{H_{\text{ext},\|} < H_{ani}\right\} \\ H_{\text{ext},\|} + M & \left\{H_{\text{ext},\|} > H_{ani}\right\} \end{matrix} \quad (25)}$$

Case when there is no perpendicular magnetic field

When there is no perpendicular external magnetic field from Eq.(16)

$$\cot\left(\alpha_{H\,\text{int}}\right) = \left(\frac{H_{ani}}{M} + 1\right)\cot\left(\alpha_M\right) \quad (30)$$

$$H_{\text{int}} = \sqrt{\left(M\frac{H_{\text{ext},||}}{H_{ani}}\right)^2 + \left(H_{ani}+M\right)^2\left[1-\left(\frac{H_{\text{ext},||}}{H_{ani}}\right)^2\right]} \quad (21)$$

or

$$H_{\text{int}} = \sqrt{\left(H_{ani}+M\right)^2 - \left(H_{ani}^2 + 2H_{ani}M\right)\left(\frac{H_{\text{ext},||}}{H_{ani}}\right)^2} \quad (21a)$$

or

$$H_{\text{int}} = M\sqrt{\left(1+\frac{H_{ani}}{M}\right)^2 - \left(\frac{H_{ani}^2}{M^2} + 2\frac{H_{ani}}{M}\right)\left(\frac{H_{\text{ext},||}}{H_{ani}}\right)^2} \quad (21b)$$

when there is no internal field at all

$$H_{\text{int}} = \left(M + H_{ani}\right) \quad (22)$$

the internal field equals to the total field (See Eq. (23))

Global & local internal magnetic field

The spin-orbit magnetic field is the local magnetic field. That means that each electron experience different magnitude and direction of the spin- orbit- field.

The external magnetic field, the magnetic field, which is generated by the magnetization, and demagnetization magnetic field are global magnetic fields. That means that all electrons experience equal the global magnetic field.

In order to calculate the total global field, the spin-- orbit magnetic field should be excluded from calculation.

(fact) There is no perpendicular-to-plane of the intrinsic global magnetic field. The in-plane component equals to $M_{||} = M\frac{H_{\text{ext},||}}{H_{ani}}$

It is a good approximation to assume that the demagnetization field equal exactly to perpendicular component of the field induced by the magnetization. Then, the components global magnetic field can be calculated as

$$H_{global,\perp} = H_{ext,\perp}$$
$$H_{global,\parallel} = M_{\parallel} + H_{ext,\parallel} \qquad (40)$$

Substitution of Eq.(6) into Eq.(40) gives

$$H_{global,\perp} = H_{ext,\perp}$$
$$H_{global,\parallel} = M\frac{H_{\mathrm{ext},\parallel}}{H_{ani}} + H_{ext,\parallel} = H_{ext,\parallel}\left(1+\frac{M}{H_{ani}}\right) \qquad (41)$$

The angle $\alpha_{gloabal}$ of the global magnetic field with respect to film normal can be calculated as

$$\cot\left(\alpha_{global}\right) = \frac{H_{global,\parallel}}{H_{global,\perp}} = \left(1+\frac{M}{H_{ani}}\right)\cot\left(\alpha_{ext}\right) \qquad (43)$$

The magnitude of global magnetic field can be calculated as

$$H_{global} = \sqrt{H^2_{global,\perp} + H^2_{global,\parallel}} = \sqrt{H^2_{ext,\perp} + H^2_{ext,\parallel}\left(1+\frac{M}{H_{ani}}\right)^2} \qquad (45)$$

$$H_{global} = H_{ext,\perp}\sqrt{H^2_{ext,\perp} + \cot^2\left(\alpha_{ext}\right)\left(1+\frac{M}{H_{ani}}\right)^2} \qquad (45)$$