The effective internal magnetic field in nanomagnet with perpendicular magnetic anisotropy (PMA)

Vadym Zayets

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Case when equilibrium magnetization of the nanomagnet is perpendicular-to-plane

(Conditions) A very strong exchange interaction between all localized electrons. As a result, there are no magnetic domains and all spins are parallel each other

(used Fact) localized electrons at interface and in the bulk experience different magnetic field of the spin-orbit interaction. However, their spins are strongly bound together and all spin can be considered as one magnetic object, which experience one common effective field, which is called the internal magnetic field no magnetic domains and all spins are parallel each other

(used Fact) localized electrons at interface and in the bulk experience different magnetic field of the

spin-orbit interaction. However, their spins are strongly

(object): a nanomagnet, which easy axis is perpendicular-to-plane.

The magnetization is aligned along the direction of the total magnetic field, which is some of the internal and external magnetic field $H_{total} = H_{int} + H_{ext}$. Therefore, magnetization angle α_M with respect to film normal can be calculated as g ine direction of the total magnetic field, which is some of the $H_{total} = H_{int} + H_{ext}$. Therefore, magnetization angle α_M with respect
- (1)
xternal magnetic field) $\alpha_M = 0$
agnitude of the magnetization can be calculated

$$
\tan(\alpha_{M}) = \frac{M_{\parallel}}{M_{\perp}} = \frac{H_{\text{ext,}\parallel} + H_{\text{int,}\parallel}}{H_{\text{ext,}\perp} + H_{\text{int,}\perp}} \quad (1)
$$

in equilibrium (in absence of external magnetic field) $\alpha_M=0$ Taking into account that the magnitude of the magnetization can be calculated as

$$
M^2 = M_\perp^2 + M_\parallel^2 \quad (2)
$$

The $Eq.(1)$ is simplified as

$$
\frac{\frac{M_{\parallel}}{M}}{\sqrt{1-\left(\frac{M_{\parallel}}{M}\right)^2}} = \frac{H_{\text{ext,}\parallel} + H_{\text{int,}\perp}}{H_{\text{ext,}\perp} + H_{\text{int,}\perp}} \quad (3)
$$

It assumed that the nanomagnet is sufficiently wide (in comparison to its thickness), so

there is neither demagnetization field nor the spin-orbit field in the in--plane direction. As a result, the in-plane internal magnetic field has only one contribution of magnetic field induced by the magnetization, which is just the magnetic field along the magnetization: Final magnetic field has only one contribution of magnetic
tization, which is just the magnetic field along the
stitution, which is just the magnetic field along the
stitution with the magnetic field along the
stitution o

$$
H_{\text{int},\parallel} = M_{\parallel} \quad (4)
$$

Substitution of Eq.(4) into (3) gives

$$
\frac{\frac{M_{\parallel}}{M}}{\sqrt{1-\left(\frac{M_{\parallel}}{M}\right)^2}} = \frac{H_{\text{ext,}\parallel} + M_{\parallel}}{H_{\text{ext,}\perp} + H_{\text{int},\perp}} \quad (5)
$$

The in--plane component of magnetization M_{\parallel} is calculated from known (measured) anisotropy field Hani. The feature of PMA and spin--orbit interaction is that the in--plane component of the magnetization linearly depends on the in--plane external magnetic field (see here) and the proportionality coefficient is defined as the anisotropy field: $\begin{aligned} &M_{\text{int,II}} = M_{\parallel} \quad \text{(4)}\\ & \frac{M_{\perp}}{M} = \frac{H_{\text{ext,II}}+M_{\parallel}}{H_{\text{ext,II}}+H_{\text{int,II}}} \quad \text{(5)}\\ & -\left(\frac{M_{\parallel}}{M}\right)^2 = \frac{H_{\text{ext,II}}+H_{\text{int,II}}} {H_{\text{ext,II}}+H_{\text{int,II}}} \quad \text{(5)}\\ & \text{in}\cdot \mathbf{m} \text{-plane component of magnetic field.} \end{aligned}$ The feature of PMA and spin-orbi $=\frac{H_{\text{ext,||}}+M_{||}}{H_{\text{ext,||}}+H_{\text{int,||}}}$ (5)

e component of magnetization M_{||} is calculated from known (measured)

field H_{ani}. The feature of PMA and spin-orbit interaction is that the

nponent of the magnetization li α relationships the internal magnetic monotonality coefficient is defined as the anisotropy
ives the in-plane component of the internal magnetic
ives

$$
\frac{M_{\parallel}}{M} = \frac{H_{\text{ext},\parallel}}{H_{\text{ani}}} \quad (6)
$$

Substitution of Eq.(6) into Eq.(4) gives the in-plane component of the internal magnetic field as:

$$
H_{\text{int},\parallel} = M \frac{H_{\text{ext},\parallel}}{H_{\text{ani}}} \quad (4a)
$$

Substitution of Eq.(6) into Eq.(5) gives

$$
\frac{H_{\text{ext,||}}}{H_{\text{ani}}} = \frac{H_{\text{ext,||}} + M \frac{H_{\text{ext,||}}}{H_{\text{ani}}}}{H_{\text{ext,}\perp} + H_{\text{int},\perp}} \quad (7)
$$

From Eq.(7) the perpendicular component of the internal magnetic field is found as

$$
H_{\text{int},\perp} = \frac{H_{\text{ext},\parallel} \left(1 + \frac{M}{H_{\text{ani}}}\right)}{H_{\text{unit}}} \sqrt{1 - \left(\frac{H_{\text{ext},\parallel}}{H_{\text{ani}}}\right)^2} - H_{\text{ext},\perp} \quad (8)
$$

or

$$
H_{\text{int},\perp} = \left(H_{\text{ani}} + M\right) \sqrt{1 - \left(\frac{H_{\text{ext},\parallel}}{H_{\text{ani}}}\right)^2} - H_{\text{ext},\perp} \quad (10)
$$

The angle α_{int} of the internal magnetic field with respect to film normal can be calculated as

or

$$
H_{\text{int},\perp} = (H_{\text{ani}} + M) \sqrt{1 - \left(\frac{H_{\text{ext},||}}{H_{\text{ani}}}\right)^2} - H_{\text{ext},\perp} \quad (10)
$$

The angle α_{int} of the internal magnetic field with respect to film normal can be calculated as

$$
H_{int\text{int}} = \frac{H_{ext}}{H_{int}} \left(1 + \frac{M}{H_{out}}\right) \sqrt{1 - \left(\frac{H_{ext}}{H_{out}}\right)^2} - H_{ext\perp} \quad (8)
$$
\nor\n
$$
H_{int\text{int}} = (H_{out} + M) \sqrt{1 - \left(\frac{H_{ext}}{H_{out}}\right)^2} - H_{ext\perp} \quad (10)
$$
\nor\n
$$
H_{int\text{int}} = (H_{out} + M) \sqrt{1 - \left(\frac{H_{ext}}{H_{out}}\right)^2} - H_{ext\perp} \quad (10)
$$
\nThe angle α_{int} of the internal magnetic field with respect to film normal can be calculated as\n
$$
\cot(\alpha_{H\text{int}}) = \frac{1}{\tan(\alpha_{H\text{int}})} = \frac{H_{int\perp}}{H_{int\parallel}} = \frac{(H_{out} + M) \sqrt{1 - \left(\frac{H_{ext\parallel}}{H_{out}}\right)^2} - H_{ext\perp}}{H_{int\parallel}} \quad (11)
$$
\nSubstitution of Eq.(4a) into (11) gives\n
$$
\cot(\alpha_{H\text{int}}) = \frac{(H_{out} + M) \sqrt{1 - \left(\frac{H_{ext\parallel}}{H_{out}}\right)^2} - H_{ext\perp}}{M \frac{H_{ext\parallel}}{H_{int}}} \quad (12)
$$
\nor\n
$$
\frac{\cot(\alpha_{H\text{int}}) = \left(1 + \frac{H_{out}}{M}\right) \sqrt{\left(\frac{H_{out}}{H_{ext\parallel}}\right)^2 - 1} - \frac{H_{ext\perp}}{H_{ext\parallel}} \frac{H_{out}}{M} \quad (12a)}{\left(\frac{H_{int}}{H_{int}}\right)^2 - 1} \quad (12a)
$$
\nThe angle α_M of the magnetization with respect to film normal can be calculated as

Substitution of Eq.(4a) into (11) gives

$$
\cot(\alpha_{Hint}) = \frac{\left(H_{ani} + M\right)\sqrt{1 - \left(\frac{H_{\text{ext,||}}}{H_{ani}}\right)^2} - H_{\text{ext,}\perp}}{M\frac{H_{\text{ext,||}}}{H_{ani}}}
$$
(12)

or

$$
\left| \cot \left(\alpha_{H \text{ int}} \right) = \left(1 + \frac{H_{\text{ani}}}{M} \right) \sqrt{\left(\frac{H_{\text{ani}}}{H_{\text{ext,}\parallel}} \right)^2 - 1} - \frac{H_{\text{ext,}\perp}}{H_{\text{ext,}\parallel}} \frac{H_{\text{ani}}}{M} \quad (12a) \right|
$$

The angle α_M of the magnetization with respect to film normal can be calculated as

$$
\cot(\alpha_{Hint}) = \frac{(H_{\text{out}} + M)\sqrt{1 - \left(\frac{H_{\text{ext,II}}}{H_{\text{out}}}\right)^2} - H_{\text{ext,II}}}{M\frac{H_{\text{ext,II}}}{H_{\text{out}}}}
$$
(12)
or

$$
\cot(\alpha_{Hint}) = \left(1 + \frac{H_{\text{out}}}{M}\right)\sqrt{\left(\frac{H_{\text{out}}}{H_{\text{ext,II}}}\right)^2 - 1} - \frac{H_{\text{ext,II}}}{H_{\text{ext,II}}}\frac{H_{\text{out}}}{M}
$$
(12a)
The angle α_{M} of the magnetization with respect to film normal can be calculated as

$$
\cot(\alpha_M) = \frac{M_{\perp}}{M_{\parallel}} = \frac{\sqrt{1 - \left(\frac{M_{\parallel}}{M}\right)^2}}{\frac{M_{\parallel}}{M_{\parallel}}} = \frac{\sqrt{1 - \left(\frac{H_{\text{ext,II}}}{H_{\text{out}}}\right)^2}}{\frac{H_{\text{ext,II}}}{H_{\text{out}}}}
$$
(14)
or

$$
\cot(\alpha_M) = \sqrt{\left(\frac{H_{\text{out}}}{H_{\text{ext,II}}}\right)^2 - 1}
$$
(15)

or

2 ext,|| $\cot(\alpha_M) = \sqrt{\frac{H_{ani}}{H}} \Big| -1 \quad (15)$ H $\left(\alpha_M\right)=\sqrt{\left(\frac{H_{_{ani}}}{H_{_{\rm ext}}}\right)^2}$ - $=\sqrt{\frac{H_{ani}}{H_{\text{ext},\parallel}}}$ -1

Comparison of (12) and (14) gives the ratio between angles α_M and α_{Hint} as

Comparison of (12) and (14) gives the ratio between angles
$$
\alpha_M
$$
 and α_{Hint} as
\n
$$
\cot(\alpha_{\text{Hint}}) = \left(1 + \frac{H_{\text{ani}}}{M}\right) \cot(\alpha_M) - \frac{H_{\text{ext}\perp}}{H_{\text{ext}\parallel}} \frac{H_{\text{ani}}}{M} \quad (16)
$$
\nor\n
$$
\cot(\alpha_{\text{Hint}}) = \left(\frac{H_{\text{ani}}}{M} + 1\right) \cot(\alpha_M) - \frac{H_{\text{ani}}}{M} \cot(\alpha_{\text{Hex}}) \quad (17)
$$

or

$$
\cot(\alpha_{H\text{int}}) = \left(\frac{H_{\text{ani}}}{M} + 1\right) \cot(\alpha_{M}) - \frac{H_{\text{ani}}}{M} \cot(\alpha_{H\text{ext}}) \quad (17)
$$

Comparison of (12) and (14) gives the ratio between angles α_M and α_{flux} as
 $\cot(\alpha_{\text{final}}) = \left(1 + \frac{H_{out}}{M}\right) \cot(\alpha_M) - \frac{H_{out}}{H_{out}} \frac{H_{out}}{M}$ (16)

or
 $\cot(\alpha_{\text{final}}) = \left(\frac{H_{out}}{M} + 1\right) \cot(\alpha_M) - \frac{H_{out}}{M} \cot(\alpha_{\text{flat}})$ (17)
 where the angle α_{Hext} of the angle of the external magnetic field, which is calculated as aparison of (12) and (14) gives the ratio between angles α_M and α_{Hint} as
 $(\alpha_{\text{Hint}}) = \left(1 + \frac{H_{\text{out}}}{M}\right) \cot(\alpha_M) - \frac{H_{\text{out}}}{H_{\text{out}}} \frac{H_{\text{out}}}{M}$ (16)
 $(\alpha_{\text{Hint}}) = \left(\frac{H_{\text{out}}}{M} + 1\right) \cot(\alpha_M) - \frac{H_{\text{out}}}{M} \cot(\alpha_{\text{start}}$,|| $\cot\left(\alpha_{\text{Hext}}\right) = \frac{H_{\text{ext},\perp}}{H} \quad (18)$ ext $H_{\scriptscriptstyle\ell}$ $\left(\alpha_{Hext}\right) = \frac{H_{ext,\perp}}{H_{ext,\parallel}}$ es the ratio between angles α_M and α_{Hint} as
 $-\frac{H_{\text{ext}\perp}}{H_{\text{ext}}}\frac{H_{\text{out}}}{M}$ (16)
 $-\frac{H_{\text{ant}}}{M}\cot(\alpha_{\text{Hext}})$ (17)

le of the external magnetic field, which is calculated as

netic field
 $(H_{\text{out}}+M)\sqrt{1-\left(\frac{H$ $\frac{\cot(\alpha_M) - \frac{H_{ani}}{M} \cot(\alpha_{Hex}) (17)}{\sinh \theta}$
 $\sinh \alpha_N = \frac{H_{ani}}{M}$ is the external magnetic field, which is calculated as

8)

and magnetic field
 $\sqrt{M_{\parallel}^2 + \left[(H_{ani} + M) \sqrt{1 - \left(\frac{H_{\text{ext,II}}}{H_{ani}} \right)^2} - H_{\text{ext,II}} \right]^2}$ (19)
 $(H_{ani}$

Magnitude of the internal magnetic field

$$
H_{int} = \sqrt{H_{int, \perp}^{2} + H_{int, \parallel}^{2}} = \sqrt{M_{\parallel}^{2} + \left[(H_{ani} + M) \sqrt{1 - \left(\frac{H_{ext, \parallel}}{H_{ani}} \right)^{2}} - H_{ext, \perp} \right]^{2}} \qquad (19)
$$

or

$$
H_{int} = \sqrt{\left(M \frac{H_{ext, \parallel}}{H_{ani}} \right)^{2} + \left[(H_{ani} + M) \sqrt{1 - \left(\frac{H_{ext, \parallel}}{H_{ani}} \right)^{2}} - H_{ext, \perp} \right]^{2}} \qquad (20)
$$

The total magnetic field
The total magnetic field is the sum of the internal and external magnetic fields.
From Eq.(4a)

$$
H_{total, \parallel} = H_{int, \parallel} + H_{ext, \parallel} = H_{ext, \parallel} \left(1 + \frac{M}{H_{ani}} \right) \qquad (20)
$$

from Eq.(10)

or

or
\n
$$
H_{int} = \sqrt{\left(M \frac{H_{ext}}{H_{ani}}\right)^{2} + \left[\left(H_{ani} + M\right)\sqrt{1 - \left(\frac{H_{ext}}{H_{ani}}\right)^{2}} - H_{ext}\right]^{2}}
$$
\n
\nThe total magnetic field
\nThe total magnetic field
\n
$$
H_{total} = H_{int, \parallel} + H_{ext, \parallel} = H_{ext, \parallel} \left(1 + \frac{M}{H_{ani}}\right) \quad (20)
$$
\nfrom Eq.(4a)
\n
$$
H_{total, \parallel} = H_{int, \parallel} + H_{ext, \parallel} = H_{ext, \parallel} \left(1 + \frac{M}{H_{ani}}\right) \quad (20)
$$
\nfrom Eq.(10)
\n
$$
H_{total, \perp} = H_{int, \perp} + H_{ext, \perp} = \left(H_{ani} + M\right)\sqrt{1 - \left(\frac{H_{ext, \parallel}}{H_{ani}}\right)^{2}} \quad (21)
$$
\nThe absolute value of the total field

The total magnetic field

The total magnetic field is the sum of the internal and external magnetic fields. From Eq.(4a)

$$
H_{\text{total},\parallel} = H_{\text{int},\parallel} + H_{\text{ext},\parallel} = H_{\text{ext},\parallel} \left(1 + \frac{M}{H_{\text{ani}}} \right) (20)
$$

from $Eq.(10)$

$$
H_{\text{total},\perp} = H_{\text{int},\perp} + H_{\text{ext},\perp} = (H_{\text{ani}} + M) \sqrt{1 - \left(\frac{H_{\text{ext},\parallel}}{H_{\text{ani}}}\right)^2} \quad (21)
$$

The absolute value of the total field

$$
H_{\text{total}} = \sqrt{H_{\text{total}\perp}^{2} + H_{\text{total}\parallel}^{2}} = \sqrt{\left(H_{\text{ani}} + M\right)^{2}\left(1 - \left(\frac{H_{\text{ext}}}{H_{\text{ani}}}\right)^{2}\right) + H_{\text{ext}}^{2}\left(1 + \frac{M}{H_{\text{ani}}}\right)^{2}} \qquad (22)
$$
\n
$$
H_{\text{total}} = \sqrt{\left(H_{\text{ani}} + M\right)^{2} - H_{\text{ext}}^{2}\left(\frac{H_{\text{ani}} + M}{H_{\text{ani}}^{2}}\right)^{2} + H_{\text{ext,left}}^{2}\left(1 + \frac{M}{H_{\text{ani}}}\right)^{2}} = \sqrt{\left(H_{\text{ani}} + M\right)^{2}}
$$
\nor

\n
$$
\frac{H_{\text{total}} = \left(H_{\text{ani}} + M\right) \quad (23)}
$$
\nAngle of the total magnetic field

\n
$$
\cot\left(\alpha_{H,\text{total}}\right) = \frac{H_{\text{total}\perp}}{H_{\text{total}\parallel}} = \frac{\left(H_{\text{ani}} + M\right)\sqrt{1 - \left(\frac{H_{\text{ext,left}}}{H_{\text{ani}}}\right)^{2}}}{H_{\text{ext,left}}\left(1 + \frac{M}{H_{\text{ani}}}\right)} = \frac{\sqrt{1 - \left(\frac{H_{\text{ext,left}}}{H_{\text{unit}}}\right)^{2}}}{H_{\text{ant}}}
$$
\nComparison with Eq/(15) gives

or

$$
H_{\text{total}} = (H_{\text{ani}} + M) \quad (23)
$$

Angle of the total magnetic field

$$
H_{\text{total}} = \sqrt{H_{\text{total,1}}^2 + H_{\text{total,1}}^2} = \sqrt{\left(H_{\text{out}} + M\right)^2 \left(1 - \left(\frac{H_{\text{eval}}}{H_{\text{out}}}\right)^2\right) + H_{\text{real}}^2 \left(1 + \frac{M}{H_{\text{out}}}\right)^2} \qquad (22)
$$
\n
$$
H_{\text{total}} = \sqrt{\left(H_{\text{out}} + M\right)^2 - H_{\text{ext,II}}^2 \left(\frac{H_{\text{out}} + M}{H_{\text{out}}^2}\right) + H_{\text{ext,I}}^2 \left(1 + \frac{M}{H_{\text{out}}}\right)^2} = \sqrt{\left(H_{\text{out}} + M\right)^2}
$$
\nor\n
$$
\frac{H_{\text{total}} = \left(H_{\text{out}} + M\right) \quad (23)}
$$
\nAngle of the total magnetic field\n
$$
\cot\left(\alpha_{H,\text{total}}\right) = \frac{H_{\text{total,II}}}{H_{\text{total,II}}} = \frac{\left(H_{\text{out}} + M\right) \sqrt{1 - \left(\frac{H_{\text{ext,II}}}{H_{\text{out}}}\right)^2}}{H_{\text{ext,II}} \left(1 + \frac{M}{H_{\text{out}}}\right)} = \frac{\sqrt{1 - \left(\frac{H_{\text{ext,II}}}{H_{\text{out}}}\right)^2}}{\frac{H_{\text{ext,II}}}{H_{\text{out}}}} = \sqrt{\left(\frac{H_{\text{ext,II}}}{H_{\text{ext,II}}}\right)^2 - 1} \quad (24)
$$
\nComparison with Eq/(15) gives\n
$$
\alpha_{H,\text{total}} = \alpha_M \quad (25)
$$
\nThe magnetization is aligned along the total magnetic field as was stated at the beginning.\n
$$
\text{Case when in plane magnetic field is greater that the anisotropy field}
$$
\nIn this case the magnetization is aligned in the in-plane direction\nTherefore the total magnetic field can be expressed as\n
$$
H_{\text{total}} = \frac{H_{\text{out}} + M \quad \left\{H_{\text{ext,II}} < H_{\text{out}}\right\}}{\left\{H_{\text{ext,II}} + M \quad \left\{H_{\text{ext,II}} > H_{\text{out}}\right\}} \quad (25)
$$

Comparison with Eq/(15) gives

$$
\alpha_{H,total} = \alpha_M \quad (25)
$$

The magnetization is aligned along the total magnetic field as was stated at the beginning.

Case when in-plane magnetic field is greater that the anisotropy field

In this case the magnetization is aligned in the in-plane direction Therefore ethe total magnetic field can be expressed as Case when in plane magnetic field is greater that the anisotropy field

In this case the magnetization is aligned in the in-plane direction

Therefore ethe total magnetic field can be expressed as
 $H_{\text{total}} = \frac{H_{\text{net}} + M}{H$

$$
\begin{vmatrix} H_{\text{total}} = \frac{H_{\text{ani}} + M}{H_{\text{ext},\parallel} + M} & \left\{ H_{\text{ext},\parallel} < H_{\text{ani}} \right\} & (25) \\ H_{\text{ext},\parallel} + M & \left\{ H_{\text{ext},\parallel} > H_{\text{ani}} \right\} \end{vmatrix}
$$

--

Case when there is no perpendicular magnetic field

When there is no perpendicular external magnetic field from Eq.(16)

$$
\cot(\alpha_{H\text{int}}) = \left(\frac{H_{\text{ani}}}{M} + 1\right) \cot(\alpha_M) \quad (30)
$$

$$
H_{int} = \sqrt{\left(M \frac{H_{ext,||}}{H_{ani}}\right)^{2} + \left(H_{ani} + M\right)^{2}\left[1 - \left(\frac{H_{ext,||}}{H_{ani}}\right)^{2}\right]}
$$
 (21)
or

$$
H_{int} = \sqrt{\left(H_{ani} + M\right)^{2} - \left(H_{ani}^{2} + 2H_{ani}M\right)\left(\frac{H_{ext,||}}{H_{ani}}\right)^{2}}
$$
 (21*a*)
or

$$
\sqrt{\left(H_{int}\right)^{2} + \left(H^{2} - H_{out}\right)\left(H_{out}\right)^{2}}
$$

or

$$
H_{\rm int} = \sqrt{\left(H_{\rm ani} + M\right)^2 - \left(H_{\rm ani}^2 + 2H_{\rm ani}M\right)\left(\frac{H_{\rm ext,||}}{H_{\rm ani}}\right)^2} \quad (21a)
$$

or

$$
H_{\rm int} = M \sqrt{\left(1 + \frac{H_{\rm ani}}{M}\right)^2 - \left(\frac{H_{\rm ani}^2}{M^2} + 2\frac{H_{\rm ani}}{M}\right)\left(\frac{H_{\rm ext,||}}{H_{\rm ani}}\right)^2} \quad (21b)
$$

when there is no internal field at all

$$
H_{\text{int}} = (M + H_{\text{ani}}) \quad (22)
$$

the internal field equals to the total field (See Eq. (23))

Global & local internal magnetic field

The spin-orbit magnetic field is the local magnetic field. That means that each electron experience different magnitude and direction of the spin- orbit- field.

The external magnetic field, the magnetic field, which is generated by the magnetization, and demagnetization magnetic field are global magnetic fields. That means that all electrons experience equal the global magnetic field.

In order to calculate the total global field, the spin-- orbit magnetic field should be excluded from calculation.

(fact) There is no perpendicular-to-plane of the intrinsic global magnetic field. The in-plane component equals to $M_{\parallel} = M \frac{H_{\text{ext,}\parallel}}{H}$ ani $M_{\parallel} = M \frac{H_{\odot}}{H}$ H $=$

It is a good approximation to assume that the demagnetization field equal exactly to

perpendicular component of the field induced by the magnetization. Then, the components global magnetic field can be calculated as perpendicular component of the field induced by the magnetization. Then, the
components global magnetic field can be calculated as
 $H_{global, \parallel} = H_{ext, \perp}$ (40)
 $H_{global, \parallel} = M_{\parallel} + H_{ext, \parallel}$ (40)
Substitution of Eq.(6) into Eq.(40) g (41)

spectrum of the magnetization. Then, the

diated as

(41)

spect to film normal can be calculated as

$$
H_{global,\perp} = H_{ext,\perp}
$$

$$
H_{global,\parallel} = M_{\parallel} + H_{ext,\parallel}
$$
 (40)

Substitution of Eq.(6) into Eq.(40) gives

perpendicular component of the field induced by the magnetization. Then, the components global magnetic field can be calculated as\n
$$
H_{global, \perp} = H_{ext, \perp}
$$
\n
$$
H_{global, \parallel} = M_{\parallel} + H_{ext, \parallel}
$$
\n(40)\nSubstitution of Eq.(6) into Eq.(40) gives\n
$$
H_{global, \perp} = H_{ext, \perp}
$$
\n
$$
H_{global, \parallel} = M \frac{H_{ext, \parallel}}{H_{ani}} + H_{ext, \parallel} = H_{ext, \parallel} \left(1 + \frac{M}{H_{ani}}\right) \left(41\right)
$$
\nThe angle α_{global} of the global magnetic field with respect to film normal can be calculated as\n
$$
\cot\left(\alpha_{global}\right) = \frac{H_{global, \parallel}}{H_{global, \perp}} = \left(1 + \frac{M}{H_{ani}}\right) \cot\left(\alpha_{ext}\right) \left(43\right)
$$
\nThe magnitude of global magnetic field can be calculated as

The angle α_{global} of the global magnetic field with respect to film normal can be calculated as

$$
\cot\left(\alpha_{\text{global}}\right) = \frac{H_{\text{global}}}{H_{\text{global},\perp}} = \left(1 + \frac{M}{H_{\text{ani}}}\right) \cot\left(\alpha_{\text{ext}}\right) \quad (43)
$$

The magnitude of global magnetic field can be calculated as

$$
H_{global,1} = H_{ew,1}
$$
\n
$$
H_{global,||} = M_{||} + H_{ew,||}
$$
\n(40)\nSubstitution of Eq.(6) into Eq.(40) gives\n
$$
H_{global,||} = H_{ew,1}
$$
\n
$$
H_{global,||} = M \frac{H_{ew,||}}{H_{aw}} + H_{ew,||} = H_{ew,||} \left(1 + \frac{M}{H_{aw}}\right) \left(41\right)
$$
\nThe angle α_{global} of the global magnetic field with respect to film normal can be calculated as\n
$$
\cot\left(\alpha_{global}\right) = \frac{H_{global,||}}{H_{global,||}} = \left(1 + \frac{M}{H_{aw}}\right) \cot\left(\alpha_{ew}\right) \left(43\right)
$$
\nThe magnitude of global magnetic field can be calculated as\n
$$
H_{global} = \sqrt{H_{global,||}^2 + H_{global,||}^2} = \sqrt{H_{ew,1}^2 + H_{w,||}^2 \left(1 + \frac{M}{H_{aw}}\right)^2} \left(45\right)
$$
\n
$$
H_{global} = H_{ext,1} \sqrt{H_{ew,1}^2 + \cot^2\left(\alpha_{ew}\right) \left(1 + \frac{M}{H_{aw}}\right)^2} \left(45\right)
$$