PMA energy. Calculation of magnetic energy experienced by a single electron.

Zayets 2018.11 (last modified 2021.11)

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Geometry: The equilibrium magnetization is perpendicular to interface. The electron experiences the demagnetization field H_{demag} and the magnetic field H_{SO} of the spin-orbit interaction only in the perpendicular- to- interface direction (z-direction). There is no H_{demag} and H_{SO} of in the in-plane direction (x-direction).

Part 1. Magnetic fields, which the electron experiences:

(*field 1*): External magnetic field, which is applied in both the perpendicular- to-interface direction H_z and in the in-plane direction H_x.

(*field 2*): Magnetic field of magnetization induced by all aligned electrons in a nanomagnet. This field can be imagined as a magnetic field inside the magnetic dipole induced by the electron spin. The average spin is a nanomagnet called the magnetization M. The magnetic field of magnetization is linearly proportional to the electron spin. In simplified units, the magnetic field induced by the spin equals to M.

(*field 3*) Demagnetization field H_{demag}. The magnetic field, which is generated at interface due to broken chain of aligned spins. The direction of H_{demag} is perpendicular- to- interface and opposite to M. The H_{demag} can be calculated as

$$H_{demag} = -k_{demag} \cdot M_z \quad (1.1)$$

where k_{demag} is the demagnetization factor. For an ideal plain film with an ideal interface $k_{demag}=1$ and there is no intrinsic field in a perpendicularly- magnetized film. However, it is an unrealistic case. In a real nanomagnet the k_{demag} is slightly smaller than 1 ($k_{demag}<1$) due to the surface roughness and the surface imperfection.

(field 4) Magnetic field H_{SO} of spin- orbit interaction, which is a relativistic magnetic field induced by an electrical field of atomic nuclear. It is directed along the orbital deformation (the z- direction in this case) and is proportional to the total magnetic field, which is applied along the x- direction.

$$H_{SO} = \hat{k}_{SO} \cdot H_{total} \quad (1.2a)$$

where $H_{total.}$ is the total applied eternal magnetic field and k_{so} is the constant of the spin-orbit interaction, which describes the strength of the spin- orbit interaction. In general, the k_{so} is a diagonal matrix, because the direction H_{SO} is always along to $H_{total.}$

$$\hat{k}_{SO} = \begin{pmatrix} k_{SO,x} & 0 & 0 \\ 0 & k_{SO,\mathrm{y}} & 0 \\ 0 & 0 & k_{SO,z} \end{pmatrix} \quad (1.2b)$$

due to the fact that the spin-orbit interaction by itself cannot break the time-inverse symmetry.

The k_{so} can be both positive, when H_{so} is directed along H_{total} , and negative, when H_{so} is directed opposite to H_{total}

The total magnetic field H_{total} is a sum of external magnetic field H and the intrinsic magnetic field induced by magnetization. Since the direction of the demagnetization field is always perpendicular to the interface (along the z- axis), the intrinsic field iis $H_{intr}=M_z-k_{demag}\, M_z$. and, therefore, the spin- orbit field H_{SO} in z- direction can be calculated as

$$H_{SO,z} = k_{SO,z} \cdot H_{total,z} = k_{SO,z} \cdot \left(H_z + H_{\mathrm{intr,z}}\right) = k_{SO,z} \cdot \left(H_z + \left(1 - k_{demag}\right) M_z\right) \quad (1.2)$$

In the direction along interface (the x-direction) , there is no demagnetization field and the intrinsic field just equals to the magnetic field M of the magnetization. he spin- orbit field H_{SO} in z- direction can be calculated as

$$H_{SO,z} = k_{SO,x} \cdot H_{total,\mathrm{x}} = k_{SO,\mathrm{x}} \cdot \left(H_x + H_{\mathrm{intr,x}}\right) = k_{SO,\mathrm{x}} \cdot \left(H_x + M_x\right) \quad (1.2)$$

In the case of zero orbital moment, the magnetic energy of an electron is a product of the magnetization M and the total magnetic field, which the electron experiences: $H+M+ H_{demag} + H_{SO}$

The magnetic energy can be calculated as

$$-Energy = \vec{H}_{all} \cdot \vec{M} = H_{all,x} M_x + H_{all,z} M_z = -\left(Energy_x + Energy_z\right) \quad (1.3)$$

Substituting Eq.(2) into Eq.(3), the magnetic energy from z- components is calculated as

$$\begin{aligned} -Energy_z &= \left(H_z + M_z + H_{demag} + H_{SO,z}\right) \cdot M_z = \\ &= \left(H_z + M_z - k_{demag} M_z + k_{SO,z} \cdot \left(H_z + \left(1 - k_{demag}\right) M_z\right)\right) \cdot M_z = \quad (1.4) \\ &= \left(1 + k_{so,z}\right) \cdot \left[H_z + \left(1 - k_{demag}\right) M_z\right] \cdot M_z \end{aligned}$$

The magnetic energy from x- components is calculated as

$$-Energy_x = \left(H_x + M_x + H_{SO,x}\right) \cdot M_x = \left(1 + k_{so,\mathrm{x}}\right)\left(H_x + M_x\right) \cdot M_x \quad (1.5)$$

Addition of Eq.(1.4) and (1.5) gives

$$-Energy = \left(1+k_{so,\mathrm{x}}\right)\left(H_x + M_x\right)\cdot M_x + \left(1+k_{so,z}\right)\left[H_z + \left(1-k_{demag}\right)M_z\right]\cdot M_z \quad (1.6)$$

Eq.(1.6) can be rewritten as

$$-Energy1 = \left(H_x + M_x\right)\cdot M_x + \left(1+k_{so}\right)\left[H_z + \left(1-k_{demag}\right)M_z\right]\cdot M_z \quad (1.6n)$$

where

$$\left(1+k_{so}\right) = \frac{\left(1+k_{so,z}\right)}{\left(1+k_{so,\mathrm{x}}\right)} \quad (1.5a)$$

$$Energy1 = \frac{Energy}{\left(1+k_{so,\mathrm{x}}\right)} \quad (1.5b)$$

Eq.(1.6n) can be simplified as

$$\boxed{-Energy1 = M^2 + H_x M_x + \left(1+k_{so}\right)H_z M_z + M_z^2\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right) \quad (1.6d)}$$

where absolute value of Magnetization, which does not change when the magnetization is tilted, is calculated as

$$M = \sqrt{M_x^2 + M_z^2} \quad (1.7)$$

(Note): The magnetic energy is negative,
(Note) The magnetic energy is of the same form when $k_{so,x}=0$ and $k_{so,z}= k_{so}$

Part 1a. Approximation and limitations of the model

(*approximation 1*) Demagnetization field in the in-plane direction (the x-direction) is ignored as if the nanomagnet is infinitely wide
(*approximation 2*) It is assumed that the demagnetization field is the same at interface and in the bulk of the nanomagnet.
(*approximation 3*) It is assumed the linear depence of the magnetic field of the spin-orbit interaction vs the total external magnetic field.

$$H_{SO} = \hat{k}_{SO}\cdot H_{total} \quad (1.2a)$$

and the non-linear dependence is ignored.

Part 2. Thickness dependence of the SO interaction

The magnetic field of the spin- orbit interaction H_{SO} is the local magnetic field and is not global. It means that each electron experiences a substantially different H_{SO} depending on the orbital symmetry and the orbital deformation. Often ever two neighbor orbitals experience substantially different H_{SO}. Usually the H_{SO} is very different for an orbital deep in bulk and for an orbital at the interface due to a different orbital deformation and orbital symmetry.

The exchange interaction strongly bounds magnetic properties of different orbitals in a nanomagnet. Then, the average or effective magnetic field of the spin-orbit interaction determines the magnetic properties of the whole nanomagnet.
Then, the nanomagnet as one object interact with the external magnetic field (SO part) as

$$H_{SO} = k_{SO} \cdot H_{total} \quad (2.0)$$

The effective constant of the spin-orbit interaction

$$k_{so} = \frac{k_{so,bulk} \cdot t + k_{so,\text{interface}} \cdot t_{so,\text{interface}}}{t + t_{so,\text{interface}}} \quad (2.1)$$

where t is the film thickness, $t_{so,interface}$ is the effective thickness at interface, where the SO is different from the bulk value; Eq.(2.1) can be further simplified as:

$$k_{so} = \frac{k_{so,bulk}\dfrac{t}{t_{so,\text{interface}}} + k_{so,\text{interface}}}{\dfrac{t}{t_{so,\text{interface}}} + 1} = k_{so,bulk} + \frac{k_{so,\text{interface}} - k_{so,bulk}}{\dfrac{t}{t_{so,\text{interface}}} + 1} \quad (2.1a)$$

From Eq. (2.1a) the coefficient of the SO interaction can be expressed as

$$\boxed{k_{so} = k_b + \frac{k_i}{t_r} \quad (2.2)}$$

where the relative film thickness

$$t_r = \frac{t}{t_{so,\text{interface}}} + 1 \quad (2.3)$$

and k_i is the difference between SO coefficient at the interface relatively to the bulk SO coefficient

$$k_i = k_{so,\text{interface}} - k_{so,bulk} \quad (2.4)$$

and k_b is the SO coefficient in the bulk

$$k_b = k_{so,bulk} \quad (2.5)$$

Part 3. Equilibrium state. There is no an external magnetic field. Critical thickness for equilibrium magnetization direction.

In the case H=0, the magnetic energy is calculated from Eq.(1.b) as

$$-Energy = M^2 + M_z^2\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right) \quad (3.1)$$

There are two possible cases:
(*case 1*) Equilibrium magnetization is in-plane
That is the case when

$$\left(1+k_{so}\right)\left(1-k_{demag}\right)-1<0 \quad (3.3)$$

and the minimum energy corresponds to

$$M_z = 0 \quad (3.4)$$

Since

$$0<1-k_{demag}<1 \quad (3.3a)$$

(3.3) gives

$$\left(1+k_{so}\right)<\frac{1}{1-k_{demag}} \quad (3.3b)$$

From (3.3b), the condition for the in- plane equilibrium magnetization is

$$\boxed{k_{so}<\frac{k_{demag}}{1-k_{demag}} \quad (3.3b)}$$

The case of the in-plane equilibrium magnetization is when k is whether negative or positive, but small

(*case 2*) Equilibrium magnetization is perpendicular-to-plane

That is the case when

$$k_{so}-k_{demag}-k_{demag}k_{so}>0 \quad (3.5)$$

and the minimum energy corresponds to

$$M_z = M \quad (3.6)$$

From (3.5) the condition for the perpendicular- to- plane equilibrium magnetization is

$$\boxed{k_{so}>\frac{k_{demag}}{1-k_{demag}} \quad (3.7)}$$

The case of the perpendicular- to -plane equilibrium magnetization is when k is positive and larger than

The critical thickness $t_{critical}$, at which the equilibrium magnetization changes from the in-plane to the perpendicular-to-plane direction, can be calculated from condition

$$k_{so}=\frac{k_{demag}}{1-k_{demag}} \quad (3.7a)$$

The SO coefficient, at which equilibrium direction is change from in-plane to perpendicular-to-plane direction, is defined as the critical SO coefficient

$$\boxed{k_{so,c}=\frac{k_{demag}}{1-k_{demag}} \quad (3.7b)}$$

Substitution of Eq.(2.2) into Eq.(3.7a) gives

$$k_b+\frac{k_i}{t_{critical}}=\frac{k_{demag}}{1-k_{demag}} \quad (3.8)$$

where $t_{critical}$ is the critical relative thickness, at which equilibrium direction is change from in-plane to perpendicular-to-plane direction,

From Eq.(3.8) the critical thickness can be calculated as

$$\boxed{t_{r,critical}=\frac{k_i}{\frac{k_{demag}}{1-k_{demag}}-k_b} \quad (3.10)}$$

when there is no bulk anisotropy k_b=0

$$t_{critical}=\frac{k_i\left(1-k_{demag}\right)}{k_{demag}} \quad (3.11)$$

Also, Eq.(10) gives the relation

$$\frac{k_{demag}}{1-k_{demag}} = k_b + \frac{k_i}{t_{critical}} \quad (3.10a)$$

Part 4: PMA. There is no perpendicular external magnetic field H_z =0

PMA: Perpendicular magnetic Anisotropy
Case of a nanomagnet, the equilibrium magnetization of which is perpendicular to the plane.
In this case the magnetic energy is calculated as

$$-Energy1 = M_z^2 + M_x^2 + H_x M_x + M_z^2\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right) \quad (4.1)$$

Under an in-plane magnetic field the magnetization M does not change its magnitude, but only it changes the direction. Therefore, $M^2 = M_z^2 + M_x^2$ is a constant and independent of H_x

Eq.(4.1) can be written as

$$-Energy1 = M^2 + H_x M_x + \left(M^2 - M_x^2\right)\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right) \quad (4.2)$$

The equilibrium magnetization direction corresponds to the minimum magnetic energy. Minimizing the energy with respect to M_x gives

$$0 = \frac{\partial Energy1}{\partial M_x} = H_x - 2M_x\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right) \quad (4.3)$$

The solution of Eq.(4.3) is

$$\boxed{\frac{M_x}{M} = \frac{H_x}{H_{ani}} \quad (4.4)}$$

where the anisotropy field H_{anis} is calculated as

$$\boxed{H_{anis} = 2M\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right) \quad (4.5)}$$

(*physical meaning of anisotropy field*): Anisotropy field is the in-plane magnetic filed, at which magnetization is fully turns in plane.

(*important property of anisotropy field*): Eq.(4.4) shows that the in-plane component of magnetization M_x is linearly proportional to in-plane component H_x of magnetic field. Since a linear fitting gives a relatively high precision, a measurement of the anisotropy field is a highly reliable with a high precision.

(*property 1 of H_{ani}*) The anisotropy field H_{ani} is linearly proportional to the magnetization M.
(*property 2 of H_{ani}*) H_{ani} increases when the coefficient of spin-orbit interaction k_{SO} increases.
(*property 32 of H_{ani}*) H_{ani} decreases when the demagnetization coefficient k_{demag} increases.

Substitution of Eq.4.5 into Eq.1.6d gives magnetic energy E as

$$-E = M^2 + \frac{H_{ani} M}{2}\left(\frac{M_z}{M}\right)^2 + H_x M_x + \left(1 + k_{so}\right) H_z M_z \quad (1.6e)$$

According to condition (3.5), the anisotropy field is positive for a nanomagnet with perpendicular- to- plane equilibrium magnetization.

Eq (4.5) can be simplified as

$$H_{anis} = 2M\left(k_{so}\left(1 - k_{demag}\right) - k_{demag}\right) \quad (4.5a)$$

Substitution of Eq.(3.7b) into (4.5a) gives

$$H_{anis} = 2M \cdot k_{demag}\left(\frac{k_{so}}{k_{so,c}} - 1\right) \quad (4.5b)$$

Substitution of Eq.(3.10a) into Eq.(4.5a) gives the dependence of the anisotropy field on the critical thickness as

$$H_{anis} = 2M\left(1 - k_{demag}\right)\left(k_{so} - k_b + \frac{k_i}{t_{r,critical}}\right) \quad (4.5a)$$

$$H_{anis} = 2M\left(\left(k_b + \frac{k_i}{t_r}\right)\left(1 - k_{demag}\right) - k_{demag}\right) \quad (4.6)$$

Substituting (3.8) into (4.6) gives the thickness dependence of the anisotropy field

$$H_{anis} = 2M\left(1 - k_{demag}\right)\left(\left(k_b + \frac{k_i}{t_r}\right) - \left(k_b + \frac{k_i}{t_{r,critical}}\right)\right) \quad (4.7)$$

Simplification of Eq.(4.7) gives the thickness dependency of the anisotropy field as

$$\boxed{H_{anis} = 2M \cdot k_i\left(1 - k_{demag}\right)\left(\frac{1}{t} - \frac{1}{t_{r,critical}}\right)} \quad (4.8)$$

The anisotropy field increase when the nanomagnet thickness decreases. The increase is linearly proportional to 1/thickness.

From an experimental measurement of anisotropy field vs. 1/t, the critical thickness can be found.

Part 5. Energy of Perpendicular Magnetic Anisotropy (PMA)

From Eq. (4.5)

$$\frac{H_{anis}}{2M} = \left(1 - k_{demag}\right)\left(1 + k_{so}\right) - 1 \quad (5.1)$$

Substitution of Eq.(5,1) into Eqs.(1.4) gives the energy as

$$-Energy_z = M_z^2 + \left[\left(1+k_{so}\right) H_z + \frac{H_{anis}}{2}\frac{M_z}{M} \right] \cdot M_z$$

$$-Energy_x = \left(H_x + M_x\right) \cdot M_x \qquad (5.2)$$

$$Energy = Energy_x + Energy_z$$

or

$$-Energy = M^2 + H_x M_x + \left(1+k_{so}\right) H_z \cdot M_z + \frac{H_{anis}}{2}\frac{M_z^2}{M} \quad (5.3)$$

When there is no external magnetic field H_x=0; H_z=0, the magnetic energy is calculated from Eq.(5.3) as

$$Energy = M^2 + \frac{H_{anis} M}{2} \quad (5.4)$$

If we can switch off the spin- orbit interaction k_{SO}=0 and demagnetization field k_{demag}=0 (just imagine it), H_{SO} becomes zero and the magnetic energy is calculated as

$$Energy = M^2 \quad (5.5)$$

The PMA energy E_{PMA} is defined as an energy, which is induced by the spin-orbit interaction minus demagnetization. Comparison of Eqs.(12) and 813) gives the PMA energy as

$$\boxed{E_{PMA} = \frac{H_{anis} M}{2} \quad (5.6)}$$

(note) The anisotropy field is the parameter, which characterizes the PMA energy.

Another way to define the PMA energy: (less correct)

The PMA energy can be also defined as an energy, which is required in order to turn the magnetization fully in-plane.

The external in-plane magnetic field, which is required to turn the magnetization into the in-plane direction, equals to the anisotropy field.) Therefore in this case H_x=H_{anis} M_x=M M_z=0 and from Eq.(5.3) the magnetic energy becomes

$$Energy = M^2 + H_{anis} M \quad (5.7)$$

The difference between magnetic energies Eq.(5.4) and Eq. (5.7) is

$$dEnergy = \frac{H_{anis} M}{2} \quad (5.8)$$

which is exactly equals to the PMA energy Eq.(5.6)
It means that the difference of magnetic energies for case when magnetization is in-plane and perpendicular to plane can be calculated as the energy spent by applying H_x:. The same difference can be calculated by the integration:

$$E_{PMA} = \int_0^{H_{anis}} \vec{M} \cdot d\vec{H} = \int_0^{H_{anis}} M_x \cdot dH_x \quad (5.9)$$

The integration (5.9) often used to evaluate the PMA energy from measured dependence of Mx vs Hx.

(Note) Calculation of E_{PMA} for more complex cases by this method may give a systematic error (See details here:). It is much more reliable to use the 1st method when E_{PMA} is calculated simply by setting $k_{SO}=0$ and $k_{demag}=0$

Part 6: PMA. There is a external magnetic field in both the perpendicular-to-plane and in-plane directions $H_z \neq 0$ $H_x \neq 0$

Case of a nanomagnet, the equilibrium magnetization of which is perpendicular to the plane.

Substitution of From Eq.(4.5) into Eq.(1.6d) gives the magnetic energy as

$$-Energy1 = M^2 + H_x M_x + (1+k_{so}) H_z \cdot M_z + \frac{H_{anis}^{(0)}}{2}\frac{M_z^2}{M} \quad (6.1)$$

where $H_{anis}^{(0)}$ is the anisotropy field in absence of the perpendicular magnetic field $H_z=0$, which is calculated by Eq. (4.5).

Under an external magnetic field the magnetization M may change direction, but it does not change its magnitude. Therefore, $M = M_z^2 + M_x^2$ is a constant and independent of H_z and H_x. Then, Eq.(6.1) is simplified as

$$Energy = M^2 + H_x M_x + \frac{H_{anis}^{(0)}}{2}\frac{M^2 - M_x^2}{M} + (1+k_{so}) H_z \sqrt{M^2 - M_x^2} \quad (6.2)$$

The equilibrium magnetization direction corresponds to the minimum magnetic energy. Minimizing the energy with respect to M_x gives the condition

$$0 = \frac{\partial Energy}{\partial M_x} = H_x - H_{anis}^{(0)}\frac{M_x}{M} - (1+k_{so}) H_z \frac{2M_x}{2\sqrt{M^2 - M_x^2}} \quad (6.3)$$

The solution of Eq.(6.3) is

$$\frac{H_x}{H_{anis}^{(0)}} = \frac{M_x}{M}\left[1 + (1+k_{so})\frac{M}{\sqrt{M^2 - M_x^2}}\frac{H_z}{H_{anis}^{(0)}}\right] \quad (6.4)$$

Even though the dependence M_x vs H_x deviates from linear, it is close to a linear dependence and can be expressed similar to Eq.(4.4) as

$$\boxed{\frac{H_x}{H_{anis}^{(H_z)}} = \frac{M_x}{M}} \quad (6.5)$$

in which the anisotropy field depends on H_z and can be calculated as

$$H_{anis}^{(H_z)} = H_{anis}^{(0)} \left[1 + \frac{1+k_{so}}{\sqrt{1-\left(\frac{M_x}{M}\right)^2}} \frac{H_z}{H_{anis}^{(0)}} \right] \quad (6.6)$$

Substitution Eq.(6.5) into Eq. (6.6) gives

$$\boxed{H_{anis}^{(H_z)} = H_{anis}^{(0)} \left[1 + \frac{1+k_{so}}{\sqrt{1-\left(\frac{H_x}{H_{anis}^{(H_z)}}\right)^2}} \frac{H_z}{H_{anis}^{(0)}} \right] \quad (6.7)}$$

When the applied in-plane magnetic field is much smaller than the anisotropy field

$$H_x << H_{anis}^{(H_z)} \quad (6.8)$$

Eq.(6.7) becomes independent of H_x and is simplified to

$$H_{anis}^{(H_z)} = H_{anis}^{(0)} \left[1 + (1+k_{so}) \frac{H_z}{H_{anis}^{(0)}} \right] \quad (6.9)$$

When condition (6.8) is not the case and the non-linear contribution of the dependence M_x vs H_x (Eqs. (6.5,6.6)) is substantial, Eq. (6.7)) can be solved by an iteration starting from Eq.(6.9)

From Eq (6.9), the anisotropy field under perpendicular magnetic field H_z can be express as

$$\boxed{H_{anis}^{(H_z)} - H_z = H_{anis}^{(0)} + k_{so} \cdot H_z \quad (6.10)}$$

where

$$H_{anis}^{(0)} = 2M\left((1+k_{so})(1-k_{demag})-1\right) \quad (4.5)$$

is anisotropy field when H_z=0.

From a linear fitting, an experimental measurement of H_{anis}-H_z, both $H_{anis}^{(0)}$ and k_{so} can be evaluated

Substitution Eq.(2.2) into Eq.(6.10) gives the nanomagnet-thickness dependence of the anisotropy field as

$$H_{anis}^{(H_z)} - H_z = H_{anis}^{(0)} + \left(k_b + \frac{k_i}{t}\right) \cdot H_z \quad (6.11)$$

When the magnetization M, the coefficient of spin-orbit interaction and the anisotropy field are measured experimentally, the demagnetization factor can be evaluated from Eq. (4.5) as:

$$k_{demag} = 1 - \frac{1 + \frac{H_{anis}^{(0)}}{2M}}{1 + k_{so}} \quad (6.12)$$

Effective internal field

Even in absence of the external magnetic field H_z, there is an intrinsic perpendicular – to- plane magnetic field $H_{z,int}$, which holds the magnetization along the easy axis in the equilibrium. Then, the anisotropy field can be calculated similarly to Eq.(6.10) as:

$$H_{anis}^{(H_z)} = (1 + k_{so})(H_z + H_{z,\text{int}}) \quad (6.14)$$

where H_z is the external magnetic field and $H_{z,int}$ is the internal magnetic field

Comparison of Eqs.(6.10) and (6.14) gives

$$H_{anis}^{(0)} = (1 + k_{so}) \cdot H_{z,\text{int}} \quad (6.15)$$

From (6.15) the *effective internal magnetic field* $H_{z,int}$ can be evaluated as

$$\boxed{H_{z,\text{int}} = \frac{H_{anis}^{(0)}}{(1 + k_{so})} \quad (6.16)}$$

Part 7. In-plane equilibrium magnetization. There is no perpendicular external in-plane magnetic field H_x=0

Case of a nanomagnet, the equilibrium magnetization of which is in-plane
In this case the magnetic energy is calculated from Eq. (1.6d) as

$$-Energy1 = M^2 + (1 + k_{so}) H_z M_z + M_z^2 \left((1 + k_{so})(1 - k_{demag}) - 1\right) \quad (7.1)$$

The equilibrium magnetization direction corresponds to the minimum magnetic energy. Minimizing the energy Eq.(7.1) with respect to M_z gives

$$0 = \frac{\partial Energy1}{\partial M_z} = (1 + k_{so}) H_z + 2M_z \left((1 + k_{so})(1 - k_{demag}) - 1\right) \quad (7.3)$$

The solution of Eq.(7.3) is

$$\boxed{\frac{M_z}{M} = \frac{H_z}{H_{ani}} \quad (7.4)}$$

where the anisotropy field H_{anis} is calculated as

$$\boxed{H_{ani} = \frac{2M}{1 + k_{so}} \left(1 - (1 + k_{so})(1 - k_{demag})\right) \quad (7.5)}$$

or

$$H_{ani} = \frac{2M}{1 + k_{so}} \left(k_{demag} - k_{so}(1 - k_{demag})\right) \quad (7.5a)$$

Substitution of Eq.(2.2) into Eq.(7.5a) gives the thickness dependence of the anisotropy field as

$$H_{ani} = \frac{2M}{1+k_b+\frac{k_i}{t}}\left(k_{demag} - \left(k_b+\frac{k_i}{t}\right)\left(1-k_{demag}\right)\right) \quad (7.6)$$

Substituting (3.8) into (7.6) gives

$$H_{ani} = \frac{2M\left(1-k_{demag}\right)}{1+k_b+\frac{k_i}{t}}\left(\left(k_b+\frac{k_i}{t_{critical}}\right)-\left(k_b+\frac{k_i}{t}\right)\right) \quad (7.7)$$

or

$$\boxed{H_{ani} = \frac{2M\left(1-k_{demag}\right)k_i}{1+k_b+\frac{k_i}{t}}\left(\frac{1}{t_{critical}}-\frac{1}{t}\right) \quad (7.7)}$$

The anisotropy field increase with an increase of nanomagnet thickness.

When there is no any SO interaction k_{SO} =0, Eq.(7.5a) gives

$$\boxed{H_{ani} = 2M \cdot k_{demag} \approx 2M \quad (7.10)}$$

Part 8: In-plane equilibrium magnetization. There is a external magnetic field in both in-plane and perpendicular-to-plane directions $H_z \neq 0$ $H_x \neq 0$

Case of a nanomagnet, the equilibrium magnetization of which is <u>in-plane</u>

The substitution of Eq. (7.5) into Eq.(1.6d) gives

$$-Energy1 = M^2 + H_x M_x + \left(1+k_{so}\right)H_z M_z - M_z^2\left(1+k_{so}\right)\frac{H_{ani}^{(0)}}{2M} \quad (8.1)$$

where

$$H_{ani,0} = \frac{2M}{1+k_{so}}\left(1-\left(1+k_{so}\right)\left(1-k_{demag}\right)\right) \quad (7.5)$$

is the anisotropy field when H_x=0. Using $M = M_z^2 + M_x^2$ Eq(8.1) is simplified to

$$-Energy1 = M^2 + H_x\sqrt{M^2-M_z^2} + \left(1+k_{so}\right)H_z M_z - \frac{H_{ani,0}\left(1+k_{so}\right)}{2M}M_z^2 \quad (8.4)$$

The equilibrium magnetization direction corresponds to the minimum magnetic energy. Minimizing the energy with respect to M_x gives

$$0 = \frac{\partial Energy1}{\partial M_z} = -H_x\frac{2M_z}{2\sqrt{M^2-M_z^2}} + \left(1+k_{so}\right)H_z - \frac{H_{ani,0}\left(1+k_{so}\right)}{M}M_z \quad (8.5)$$

or

$$\left(1+k_{so}\right)H_z=\frac{M_z}{M}\left(H_{ani,0}\left(1+k_{so}\right)+\frac{H_x}{\sqrt{1-\left(\frac{M_z}{M}\right)^2}}\right)\quad(8.5a)$$

Solution of Eq.(8.5) can be expressed as

$$\boxed{\frac{H_z}{H_{anis}^{(H_x)}}=\frac{M_z}{M}\quad(8.6)}$$

where the anisotropy field is calculated as

$$H_{anis}^{(H_x)}=H_{ani,0}+\frac{H_x}{\left(1+k_{so}\right)\sqrt{1-\left(\frac{M_z}{M}\right)^2}}\quad(8.7)$$

Substitution of Eq.(8.6) into (8.7) gives

$$\boxed{H_{anis}^{(H_x)}=H_{ani,0}+\frac{H_x}{\left(1+k_{so}\right)\sqrt{1-\left(\frac{H_z}{H_{anis}^{(H_x)}}\right)^2}}\quad(8.7a)}$$

when the applied perpendicular- to- plane magnetic field H_z is

$$H_z\ll H_{anis}^{(H_x)}\quad(8.8)$$

The anisotropy field is independent of Hz

$$\boxed{H_{anis}^{(H_x)}=H_{ani,0}+\frac{H_x}{\left(1+k_{so}\right)}\quad(8.9)}$$

The anisotropy field is linearly proportional to the applied external magnetic field H_x.

Part 9. Energy barrier for the magnetization switching

This is the case when a magnetic field is applied along the magnetic easy axis (perpendicularly to the film) (H_x=0 H_z=H), the energy is calculated from Eq.(1.6d) as

$$-E=M^2+\left(1+k_{so}\right)H\cdot M_z+M_z^2\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right)\quad(1.6e)$$

or

$$-E=\cos^2\left(\theta\right)\cdot\left[M^2\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right)\right]+\cos\left(\theta\right)\left[\left(1+k_{so}\right)M\cdot H\right]+M^2\quad(1.6f)$$

where θ is the angle between the magnetization M and the film normal,

Substitution of Eq.(7.5)

$$H_{ani}=\frac{2M}{1+k_{so}}\left(\left(1+k_{so}\right)\left(1-k_{demag}\right)-1\right)\quad(7.5)$$

into Eq.(1.6f) gives

$$-E=\cos^2\left(\theta\right)\cdot\left[\frac{H_{ani}}{2}\left(1+k_{so}\right)M\right]+\cos\left(\theta\right)\left[\left(1+k_{so}\right)M\cdot H\right]+M^2\quad(1.6g)$$

or

$$-\frac{E}{(1+k_{so})M}=\cos^2(\theta)\cdot\frac{H_{ani}}{2}+\cos(\theta)\cdot H+const \quad (1.6g)$$

The maximums and minimums of energies can be found from the condition

$$0=\frac{\partial E}{\partial\theta}\left(\frac{-E}{(1+k_{so})M}\right)=-2\sin(\theta)\cos(\theta)\cdot\frac{H_{ani}}{2}-\sin(\theta)\cdot H \quad (9.1)\text{ or}$$

The energy maximum is at the magnetization angle θ_{max}

$$\cos(\theta_{\max})=-\frac{H}{H_{ani}} \quad (9.2)$$

In the case when the external field H is substantially smaller than the anisotropy field H_{ani}, the magnetization angle of the maximum energy is closed to the hard axis direction $\theta_{max} \sim +90$ deg and -90 deg.

Substitution of Eq. (9.2) into Eq.(1.6g) gives the maximum energy as

$$-E_{\max}=\left(\frac{H}{H_{ani}}\right)^2\cdot\left[\frac{H_{ani}}{2}(1+k_{so})M\right]-\frac{H}{H_{ani}}\left[(1+k_{so})M\cdot H\right]+M^2 \quad (9.3)$$

or

$$-E_{\max}=\frac{H}{H_{ani}}(1+k_{so})\left[\frac{H}{H_{ani}}\frac{MH_{ani}}{2}-M\cdot H\right]+M^2 \quad (9.3a)$$

or

$$-E_{\max}=-\frac{H}{H_{ani}}(1+k_{so})\frac{M\cdot H}{2}+M^2 \quad (9.3b)$$

The energy minimum is at θ min=0 and 180 degrees, the minimum energy is

$$-E_{\min}=\left[\frac{H_{ani}}{2}(1+k_{so})M\right]+\left[(1+k_{so})M\cdot H\right]+M^2 \quad (9.4)$$

The energy barrier is a difference between the minim and maximum energy

$$E_{barrier}=E_{\max}-E_{\min}=\frac{H}{H_{ani}}(1+k_{so})\frac{M\cdot H}{2}+\left[\frac{H_{ani}}{2}(1+k_{so})M\right]+\left[(1+k_{so})M\cdot H\right] \quad (9.5\text{a})$$

or

$$E_{barrier}=\frac{H_{ani}M}{2}(1+k_{so})+(1+k_{so})M\cdot H\left[1+\frac{H}{2H_{ani}}\right] \quad (9.5)$$

The magnetization switching occurs at magnetic field equals to the coercive field H_c, which is substantially smaller that H_{ani}. (**H_c << H_{ani}**) In this case

$$\boxed{E_{barrier}\approx\frac{H_{ani}M}{2}(1+k_{so})+(1+k_{so})M\cdot H \quad (9.6)}$$

Note it is the energy barrier in the case when the magnetic field is along the easy axis and the magnetization direction is changing. It means that it is the case of the magnetization precession.

Part 10. Comparison of this full model with the oversimplified Neel model

L. Néel, Adv. Phys., 1955

In the Neel model, the magnetic energy is described as

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M} \right)^2 + \vec{M} \cdot \vec{H} \quad (10.1)$$

where M is the magnetization, H is external magnetic field M_z is the magnetization component along the easy axis and M_x is the magnetization component along the hard axis. Eq. 10.1 can be wrote as

$$-E = E_{PMA} \cdot \left(\frac{M_z}{M} \right)^2 + M_z H_z + M_x H_x \quad (10.1a)$$

This full model describes the magnetic energy is described as Eq. (1.6d)

$$-E = M_z^2 \left(\left(1 + k_{so}\right)\left(1 - k_{demag}\right) - 1 \right) + \left(1 + k_{so}\right) H_z M_z + H_x M_x + M^2 \quad (10.2)$$

Comparison of Eqs.(10.1a) and (10.2) gives

$$E_{PMA} = M^2 \left(\left(1 + k_{so}\right)\left(1 - k_{demag}\right) - 1 \right) \quad (10.2)$$

From comparison of the 2nd and 3rd term, it can be concluded that the oversimplified Neel model are fully identical when the coefficient of the spin-orbit interaction equals to zero:

$$k_{so} = 0 \quad (10.3)$$

It is nearly the case for a single-layer ferromagnetic nanomagnet, but it is not the case for a multilayer nanomagnet.