Spinor in a magnetic field.

Spin precession & Spin damping

Part 1. Introduction

The electron spin describes the features of the time-inverse symmetry for the electron.

When a magnetic field is applied at an angle with respect to the spin direction, the spin precession starts. Any spin precession is accompanied by a spin damping or an alignment of the spin along the applied magnetic field. The spin damping is an unavoidable process for the spin.

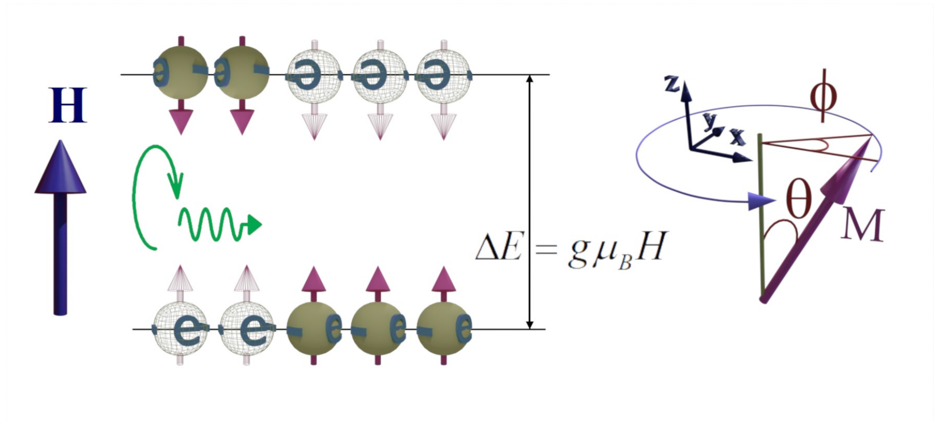

Figure 1 Two representations of the identical spin precession (left) as partially filled spin-up and spin-down

The precession damping is not a spin-conserving process and, therefore, participation of another particle with a non-zero spin (e.g. a photon, magnon etc.) is required for the precession damping to occur. In the following, the spin damping torque is calculated from a known rate of interaction of non-zero-spin particles with the magnetization.

The energy of an electron (a spin-up electron), which spin is aligned along a magnetic field H, is smaller than the energy of an electron (a spin-down electron), which spin is aligned opposite to H (See Fig.1). The energy difference is called the Zeeman energy. The magnetization precession can be represented as a quantum state, in which both the spin-down and spin-up states are partially occupied.

Part 2. Spin precession & Spinors

Absorption of a non-zero-spin particle (e.g. a photon) excites a spin-up electron to the spin-down energy level. A larger relative number of spin-down electrons corresponds to a larger precession angle θ. Therefore, the absorption of a non-zero-spin particle causes an increase of the precession angle. Similarly, the emission of a non-zero-spin particle reduces the number of spin-down electrons and, therefore, reduces the precession angle θ.

In the following, the dependence of precession angle on the number of spin-up and spin-down electrons is calculated using the spinor technique. A quantum state of an electron, which spin is aligned at angles θ and ϕ as shown in Fig.5, is described by a spinor S, which is an eigenvector of the following Pauli matrix:

$$\left[\sigma_x \cos(\phi)+\sigma_y \sin(\phi)\right]\sin(\theta)+\sigma_z \cos(\theta)= \\ =\left[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\cos(\phi)+\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\sin(\phi)\right]\sin(\theta)+\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\cos(\theta) \tag{A3.1}$$

The spinor S, which is calculated as an eigenvector of (A3.1), is

$$\hat{S}_{\theta,\phi}=\frac{1}{\sqrt{2\left(1+\cos(\theta)\right)}}\begin{pmatrix}1+\cos(\theta)\\ \sin(\theta)\cdot e^{-i\phi}\end{pmatrix} \tag{A3.2}$$

The wavefunction of an electron, which is described by the spinor (A3.2), can be expressed as linear combination of wavefunctions, which correspond to the spin -up (θ=0) and spin- down (θ=π) states or as a vector dot product:

$$\Psi_{\theta,\phi}=\hat{S}_{\theta,\phi}\cdot\left(\Psi_\uparrow \quad \Psi_\downarrow\right) \tag{A3.3}$$

The energy difference ΔE_{Zeeman} between the electron states of the spin directions along and opposite to the magnetic field H is calculated as:

$$\Delta E_{Zeeman}=g\cdot\mu_B H \tag{A3.4}$$

where g is the g- factor and μ_B is the Bohr magneton.

The wavefunction of the spin-up and spin-down electron states can be expressed as

$$\Psi_\uparrow(\vec{r},t)=\Psi(\vec{r})\cdot e^{i\frac{E_0}{\hbar}t}e^{+i\frac{g\cdot\mu_B H}{2\hbar}t} \\ \Psi_\downarrow(\vec{r},t)=\Psi(\vec{r})\cdot e^{i\frac{E_0}{\hbar}t}e^{-i\frac{g\cdot\mu_B H}{2\hbar}t} \tag{A3.5}$$

Wavefunctions of (A3.5) can be expressed using the spinor representation (A3.3) as

$$\Psi_\uparrow=\begin{pmatrix}1\\ 0\end{pmatrix}\cdot\left(\Psi_\uparrow \quad \Psi_\downarrow\right)\cdot e^{i\left(\frac{E_0}{\hbar}+\frac{g\cdot\mu_B H}{2\hbar}\right)t} \\ \Psi_\downarrow=\begin{pmatrix}0\\ e^{-i\frac{g\cdot\mu_B H}{\hbar}t}\end{pmatrix}\cdot\left(\Psi_\uparrow \quad \Psi_\downarrow\right)\cdot e^{i\left(\frac{E_0}{\hbar}+\frac{g\cdot\mu_B H}{2\hbar}\right)t} \tag{A3.6}$$

where the spin-up state corresponds to $\theta=0^0$ and the spin-down state corresponds to $\theta=180^0$. Comparison of Eqs. (A3.6) with Eq. (A3.3) gives the expression for the spinor at an arbitrary angle θ as

$$\hat{S}_{\theta,\phi} = \frac{1}{\sqrt{2\left(1+\cos(\theta)\right)}} \begin{pmatrix} 1+\cos(\theta) \\ \sin(\theta)\cdot e^{-i\frac{g\cdot\mu_B H}{\hbar}t} \end{pmatrix} \qquad \text{(A3.7)}$$

Eq.(A3.7) describes a spin precession at a precession angle θ and with the Larmor frequency ω_L:

$$\omega_L = \frac{g\cdot\mu_B H}{\hbar} \qquad \text{(A3.8)}$$

It means that the precession term of the LL equation (Eq.1) describes the Zeeman splitting and in general is a feature of the time-inverse symmetry.

The spin precession around a magnetic field can be described by a relative number of the spin-up and spin down electrons. In a simplified case when the wavefunction of the system of electrons with the spin S can be described as a sum of wavefunctions for spin-up and spin-down electrons as

$$\Psi_S = \Psi_\uparrow \cdot \sqrt{N_\uparrow} + \Psi_\downarrow \cdot \sqrt{N_\downarrow} \qquad \text{(A3.9)}$$

where $N_\uparrow$ and $N_\downarrow$ are the numbers of the spin- up and spin- down electrons. The expression (A3.9) corresponds to the spinor:

$$\hat{S} = \frac{1}{\sqrt{N_\uparrow + N_\downarrow}} \begin{pmatrix} \sqrt{N_\uparrow} \\ \sqrt{N_\downarrow} \end{pmatrix} \qquad \text{(A3.10)}$$

Comparison of Eqs. (A3.7) and (A3.10) gives

$$\sqrt{n_\uparrow} = \frac{1+\cos(\theta)}{\sqrt{2\left(1+\cos(\theta)\right)}}$$
$$\sqrt{n_\downarrow} = \frac{\sin(\theta)}{\sqrt{2\left(1+\cos(\theta)\right)}} \qquad \text{(A3.11)}$$

where $n_\uparrow$ and $n_\downarrow$ are the relative numbers of the spin up and spin down electrons

$$n_\uparrow = \frac{N_\uparrow}{N_\uparrow + N_\downarrow} \quad n_\downarrow = \frac{N_\downarrow}{N_\uparrow + N_\downarrow} \qquad \text{(A3.12)}$$

Simplification of Eq.(A3.11) gives the number of spin-up and spin-down electrons, when there is a magnetization precession and the precession angle is θ, as

$$n_\uparrow = \frac{1+\cos(\theta)}{2}$$
$$n_\downarrow = \frac{1-\cos(\theta)}{2} \qquad \text{(A3.13)}$$

Figure 2 shows the filling percentage of the spin- up and spin-down levels as a function of the precession angle. The filling amounts are the same at a precession angle of 90^0.

Part 3 Precession Damping & Precession Pumping

The probability P_{pump} to excite one electron from the spin-up to the spin-down level and, therefore, to increase the precession angle is linearly proportional to the number of the spin-up electrons and the number of available spin-down quantum states.

$$P_{pump} = P_{pump,0} \cdot n_{\uparrow} \cdot \left(1 - n_{\downarrow}\right) = P_{pump,0} \cdot \left(1 - n_{\downarrow}\right)^2 \tag{A3.14}$$

where $P_{pump,0}$ is the probability to excite a spin-up electron into one already-empty spin-down state.

Similarly, the probability that an electron returns back to the spin-up level is calculated as

$$P_{damp} = P_{damp,0} \cdot n_{\downarrow} \cdot \left(1 - n_{\uparrow}\right) = P_{damp,0} \cdot n_{\downarrow}^2 \tag{A3.15}$$

where $P_{damp,0}$ is the transition probability of a spin-down electron into an empty spin-up state. The $P_{pump,0}$ and $P_{damp,0}$ are defined by the interaction mechanism of the total spin of the localized electrons with external spin particles (magnons, phonons, spin-polarized conduction electrons).

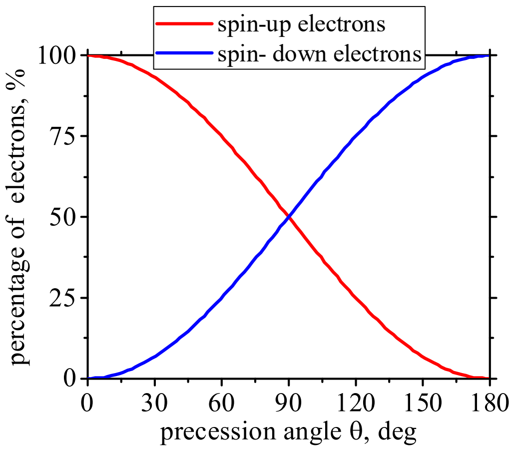

Figure 2. Percentage of spin-up and spin-down

The electron transition rate is defined as the transition probability per a unit of time. The rates of transitions between the spin-up and the spin- down levels or the pumping and damping rates can be calculated from probabilities (A3.14), (A3.15) as

$$\left(\frac{\partial n_{\uparrow}}{\partial t}\right)_{damp} = R_d \cdot n_{\downarrow}^2$$
$$\left(\frac{\partial n_{\downarrow}}{\partial t}\right)_{pump} = R_p \cdot \left(1 - n_{\downarrow}\right)^2 \tag{A3.16}$$

where R_d and R_p are the rates of the spin transition from the spin-down to the spin-up level and from the spin-down to the spin-up level, correspondingly.

The spin damping torque is calculated from (A3.13) (A3.16) as

$$\left(\frac{\partial \theta}{\partial t}\right)_{damp} = \frac{\partial \theta}{\partial n_{\downarrow}} \left(\frac{\partial n_{\downarrow}}{\partial t}\right)_{damp} = -\frac{\partial \theta}{\partial n_{\downarrow}} R_d \cdot n_{\downarrow}^2 = -R_d \frac{n_{\downarrow}^2}{\sqrt{n_{\downarrow}\left(1 - n_{\downarrow}\right)}} \tag{A3.17}$$

Similarly, the spin pumping torque is calculated as

$$\left(\frac{\partial \theta}{\partial t}\right)_{pump} = \frac{\partial \theta}{\partial n_{\downarrow}} \left(\frac{\partial n_{\downarrow}}{\partial t}\right)_{pump} = -\frac{\partial \theta}{\partial n_{\downarrow}} R_p \cdot \left(1 - n_{\downarrow}\right)^2 = R_p \cdot \frac{\left(1 - n_{\downarrow}\right)^2}{\sqrt{n_{\downarrow}\left(1 - n_{\downarrow}\right)}} \tag{A3.18}$$

The interaction of a spin particle with the magnetization of a nanomagnet also depends on the precession angle θ. For example, the emission or absorption of a circularly- polarized photon is linearly proportional to the alternating part of the magnetic moment. It is because the nanomagnet can be considered as an electro-magnetic antenna, which emits or absorbs photons, and the effectiveness of the photon absorption/ emission is proportional only to the alternating component of the magnetic moment. In the case of the precession around the z-axis, the alternating magnetic moment is promotional to the xy-component of the magnetization M (See Fig.5) and the spin pumping torque can be calculated from (A3.18) as:

$$\left(\frac{\partial\theta}{\partial t}\right)_{pump} = R_{pump}\cdot\frac{\left(1-n_{\downarrow}\right)^2}{\sqrt{n_{\downarrow}\left(1-n_{\downarrow}\right)}}\sin\left(\theta\right) \qquad \text{(A3.19)}$$

where R_{pump} is the rate of the photon absorption per unit of the alternating magnetic moment. The R_{pump} is linearly proportional to the photon flux, which irradiates the nanomagnet. The substitution of Eq.(A3. 13) into A(3.19) gives the precession pumping torque due to absorption of circular-polarized photons as

$$\left(\frac{\partial\theta}{\partial t}\right)_{pump} = R_{pump}\cdot\frac{\left(1+\cos\left(\theta\right)\right)^2\sin\left(\theta\right)}{2\sqrt{1-\cos\left(\theta\right)^2}} \qquad \text{(A3.20)}$$

Similarly, the emission of photons is also linearly proportional to the alternating magnetic moment and therefore the xy-component of the magnetization. The substitution of Eq.(A3. 13) into (A3.17) gives the precession damping torque due to emission of circular-polarized photons as

$$\left(\frac{\partial\theta}{\partial t}\right)_{damp} = -R_{damp}\frac{n_{\downarrow}^2}{\sqrt{n_{\downarrow}\left(1-n_{\downarrow}\right)}}\sin\left(\theta\right) = R_{damp}\frac{\left(1-\cos\left(\theta\right)\right)^2\sin\left(\theta\right)}{2\sqrt{1-\cos\left(\theta\right)^2}} \qquad \text{(A3.21)}$$

Figure 3 shows the spin pumping/ spin damping torque calculated from Eqs.(A3.20),(A3.21). In the absence of precession ($\theta=0^0$), there is no damping, but the pumping is substantial. With an increase of the precession angle, the damping monotonically increases and the pumping decreases.

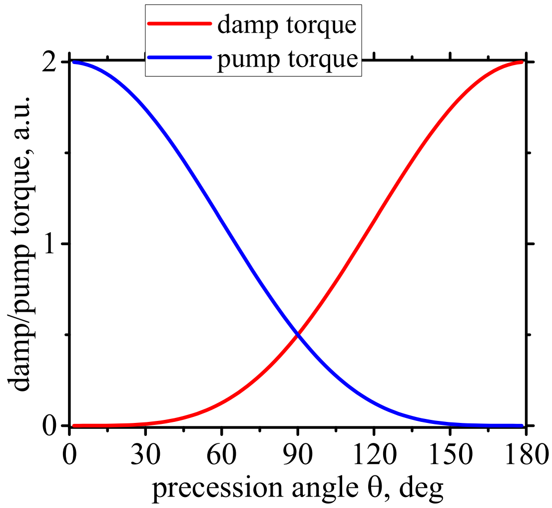

Figure 3 Precession damp/pump torque due to emission/ absorption of circularly- polarized photons.