Basic relation for the Hall effect and AMR

Hall effect

$$j_{\perp,Hall} = \alpha_{Hall} \cdot j_{\|,\text{bias}} \quad (1.1)$$

AMR effect

$$j_{\|,\text{AMR}} = \alpha_{AMR} \cdot j_{\|,\text{bias}} \quad (1.2)$$

AMR ratio vs AMR angle

AMR current depends on the magnetization direction with respect to the bias current direction.

$$\begin{aligned} j_{\|,\min} &= j_{\|,\text{bias}} - j_{\|,\text{AMR}} \\ j_{\|,\max} &= j_{\|,\text{bias}} \end{aligned} \quad (1.2)$$

$$AMR = \frac{R_{\max} - R_{\min}}{\left(R_{\max} + R_{\min}\right)/2} = 2\frac{\dfrac{V}{j_{\|,\max}} - \dfrac{V}{j_{\|,\min}}}{\dfrac{V}{j_{\|,\max}} + \dfrac{V}{j_{\|,\min}}} = \frac{1 - \dfrac{j_{\|,\min}}{j_{\|,\max}}}{1 + \dfrac{j_{\|,\min}}{j_{\|,\max}}} = \frac{\dfrac{j_{\|,\text{AMR}}}{j_{\|,\text{bias}}}}{1 - \dfrac{j_{\|,\text{AMR}}}{j_{\|,\text{bias}}}} = \frac{\alpha_{AMR}}{1 - \alpha_{AMR}} \approx \alpha_{AMR} \quad (1.2)$$

$$AMR = \frac{R_{\max} - R_{\min}}{\left(R_{\max} + R_{\min}\right)/2} = \alpha_{AMR} \quad (1.2)$$

Hall voltage vs, Hall angle

Hall current flowing perpendicularly to the wire is calculated as

$$j_{\perp,Hall} = \alpha_{Hall} \cdot j_{\|,\text{bias}} = \alpha_{Hall} \cdot \sigma \cdot \frac{V}{L} \quad (4.2)$$

where σ is the wire conductivity, ***L*** is wire length and ***V*** is applied bias voltage between ends of wire.

The Hall current creates a charge accumulation at the wall of nanowire. The charge accumulation creates a Hall voltage V_{Hall} across the width of metallic wire and a current flows across the in opposite direction to the Hall current. This balancing current is calculated as:

$$j_{\perp,balance} = \sigma \frac{V_{Hall}}{w} \quad (4.5)$$

In the equilibrium, the Hall current and teh balancing current are equal and opposite:

$$j_{\perp,balance} = j_{\perp,Hall} \quad (4.4)$$

Substitution of Eqs into Eq. gives

$$\sigma \frac{V_{Hall}}{w} = \alpha_{Hall} \cdot \sigma \cdot \frac{V}{L} \quad (4.2)$$

or

$$\alpha_{Hall} = \frac{V_{Hall}}{V} \frac{L}{w} \quad (4.7)$$

Hall resistance vs Hall angle

The Hall resistance is defined as a ratio of the Hall voltage to the bias current

$$R_{Hall} = \frac{V_{Hall}}{J_{\parallel}} \quad (4.8)$$

the Ohm's law reads

$$J_{\parallel} = \sigma \frac{V}{L} \cdot w \cdot thick \quad (4.9)$$

where *thick* is the thickness of the metallic wire

$$R_{Hall} = \frac{V_{Hall}}{V} \frac{L}{\sigma \cdot w \cdot thick} = \alpha_{Hall} \frac{1}{\sigma \cdot thick} \quad (4.10)$$

Therefore, the Hall resistance can be calculated from

$$R_{Hall} = \alpha_{Hall} \frac{1}{\sigma \cdot thick} \quad (4.10)$$
